

# Power Systems Basics

Keith Moffat

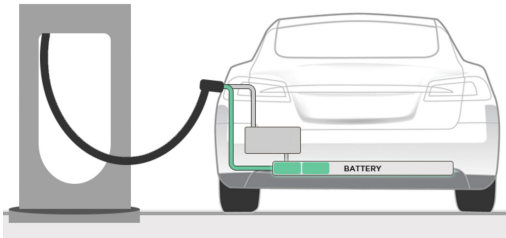
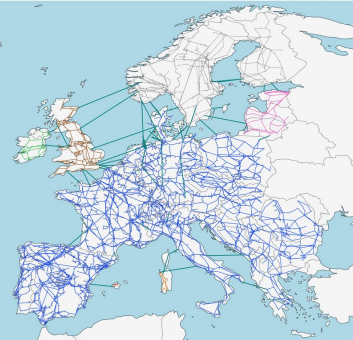
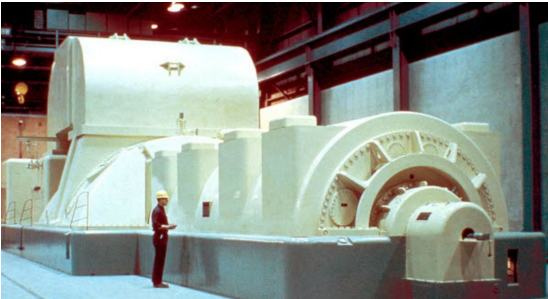
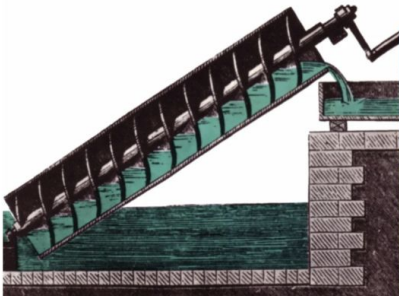
“Any time a talk says power systems in the title, I just kinda tune out.”

- anonymous IFA member

## Learning objective

Provide context for power systems research by giving an overview of power systems basics.

# An electricity analogy

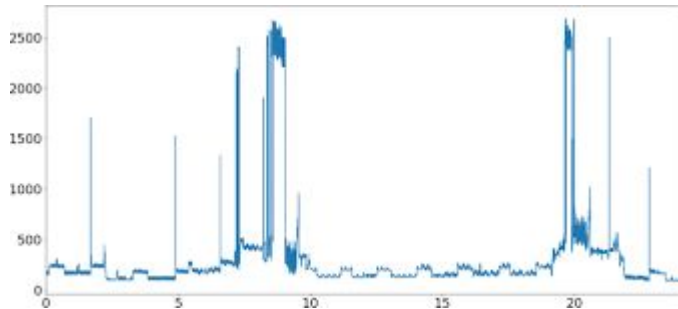


## An electricity analogy



What aspects of modern electric power does this analogy not capture well?

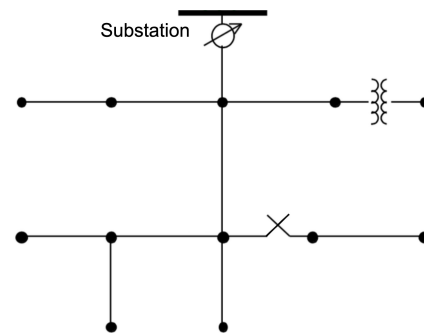
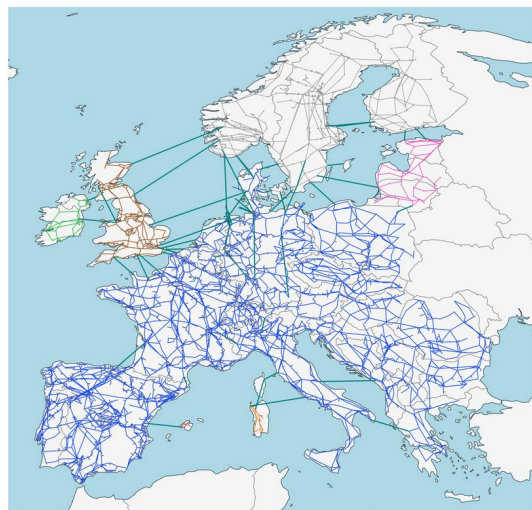
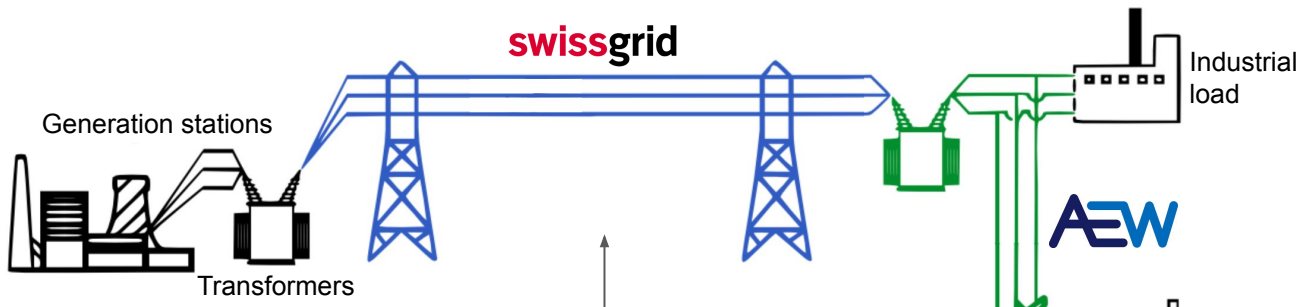
# Why do we have electric grids?



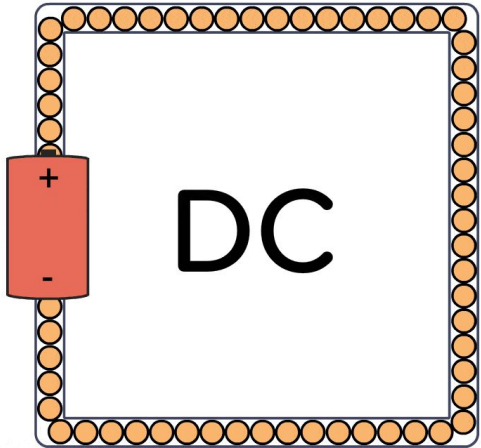
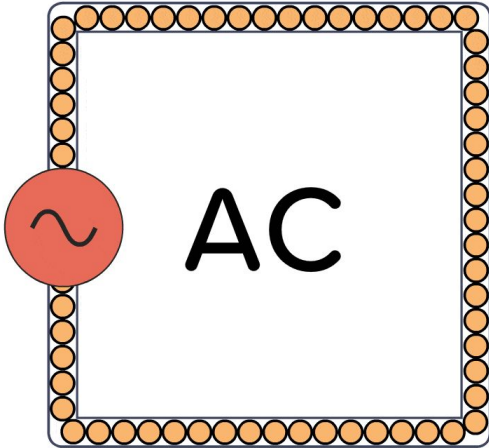
# Generation

# Transmission

# Distribution

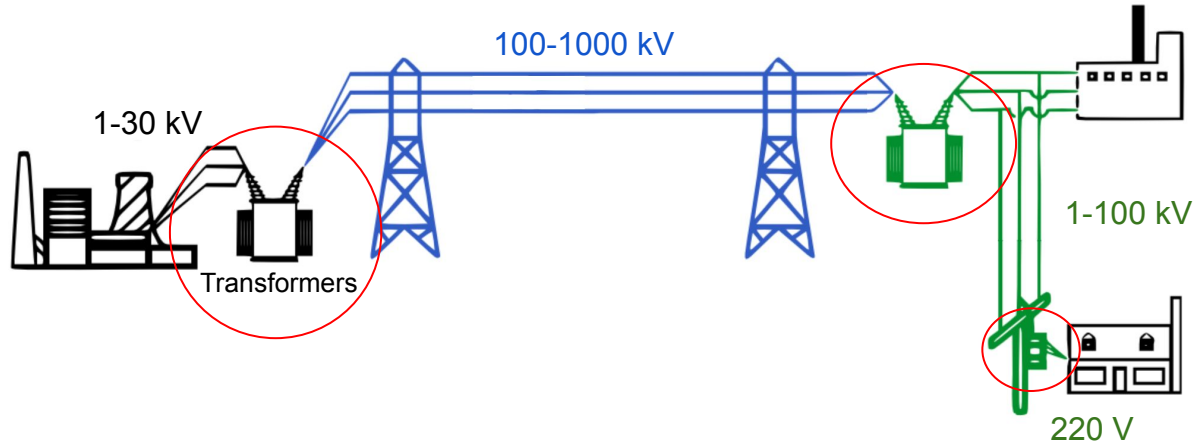


# AC vs. DC

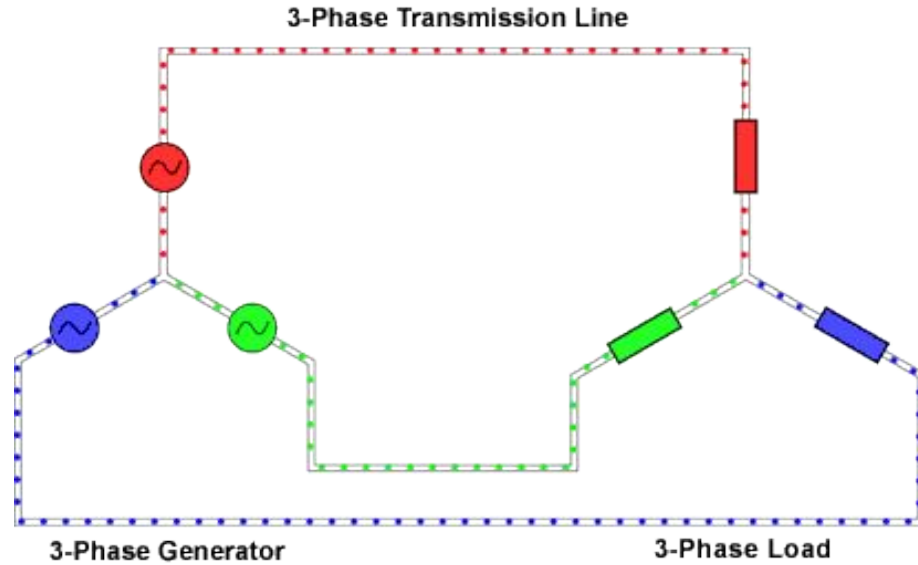
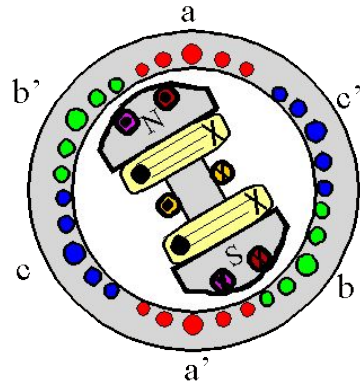




Why are grids AC rather than DC?



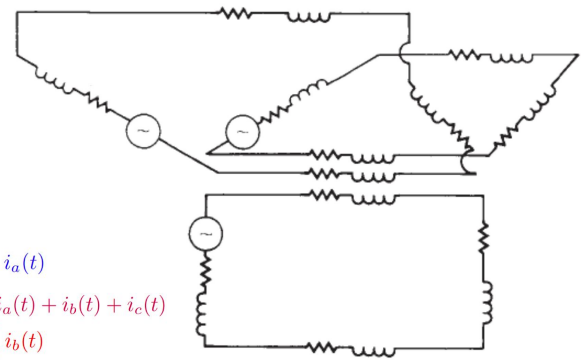
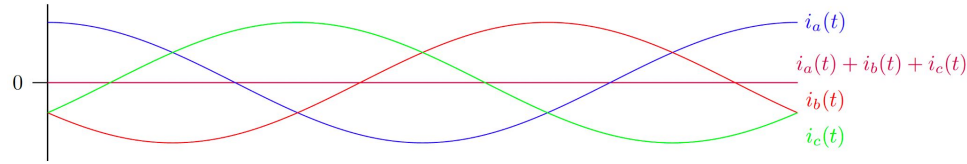
# 3-phase AC power



# Why 3 phases?

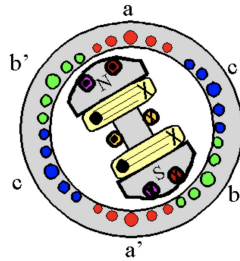
## Reason 1:

Compared with separate single-phase systems, 3-phase systems require half the number of conductors



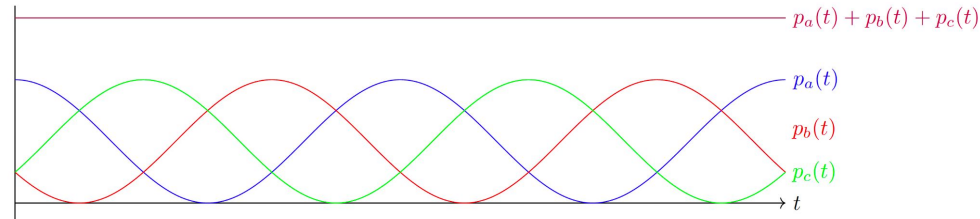
## Reason 2:

3-phase systems create a rotating magnetic field



## Reason 3:

3-phase systems deliver constant instantaneous power



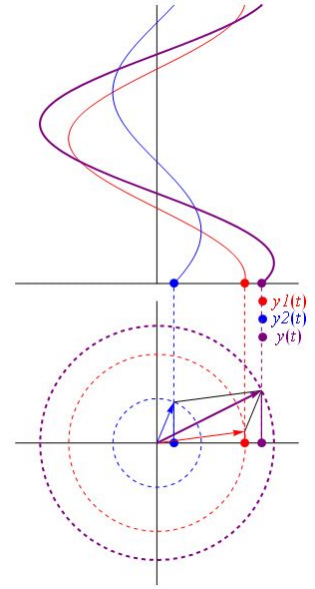
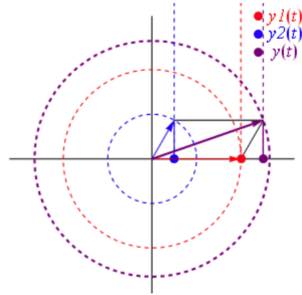
# Rotating reference frames

$$\theta(t) = \omega t + \delta$$

Time-domain signal:

$$A \cos(\omega t + \delta) = \text{Real}(Ae^{j(\omega t + \delta)})$$

Phasor representation:  $Ae^{j\delta}$



Mathematical rigor:

The phasor transformation is the Fourier Transformation of the analytic version of a single frequency signal.

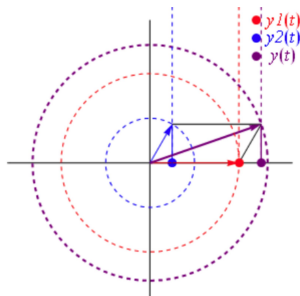
## Rotating reference frames

$$\theta(t) = \omega t + \delta$$

Time-domain signal:

$$A \cos(\omega t + \delta) = \text{Real}(Ae^{j(\omega t + \delta)})$$

Phasor representation:  $Ae^{j\delta}$



dqz representation/transformation:  $x_{dqz}(t) = T_{\theta(t)} x_{abc}(t)$

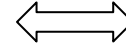
(similar but different)

$$T_{\theta(t)} = \frac{2}{3} \begin{bmatrix} \cos(\theta(t)) & \cos(\theta(t) - \frac{2\pi}{3}) & \cos(\theta(t) + \frac{2\pi}{3}) \\ -\sin(\theta(t)) & -\sin(\theta(t) - \frac{2\pi}{3}) & -\sin(\theta(t) + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

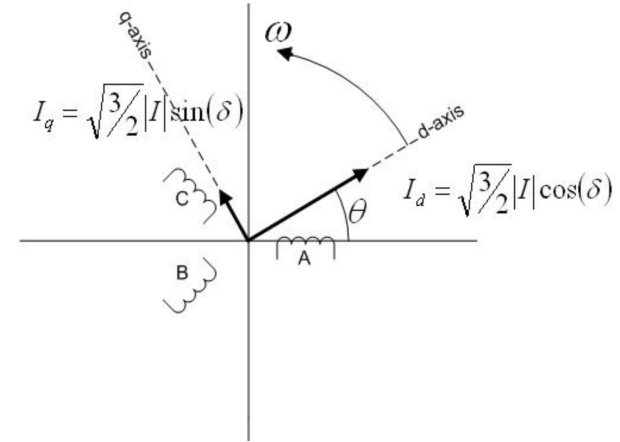
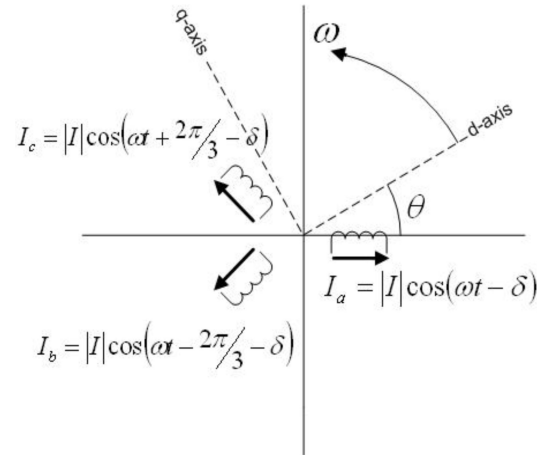
# Rotating reference frames

dqz transformation:

a/b/c signals  
(time domain,  
stationary ref. frame)



d/q/z signals  
(time domain,  
rotating ref. frame)

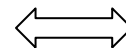


primarily used for inverter or machine control

# Rotating reference frames

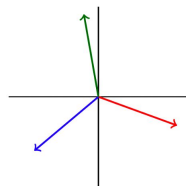
Sequence transformation:

a/b/c phasors  
(phasor domain,  
rotating ref. frame)

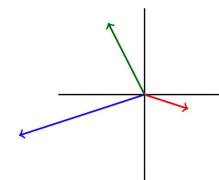


+/-/0 sequences  
(phasor domain,  
rotating ref. frame)

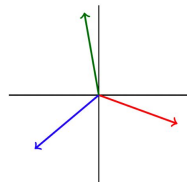
primarily used for analyzing 3-phase unbalanced signals:



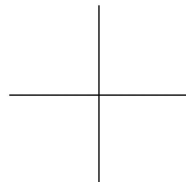
Balanced 3-phase signal



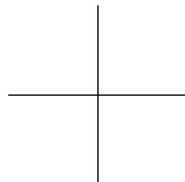
Unbalanced 3-phase signal



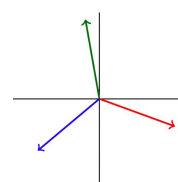
Positive Sequence



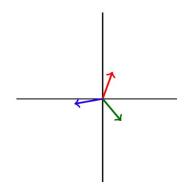
Negative Sequence



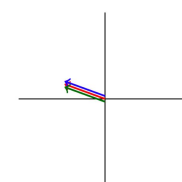
Zero Sequence



Positive Sequence



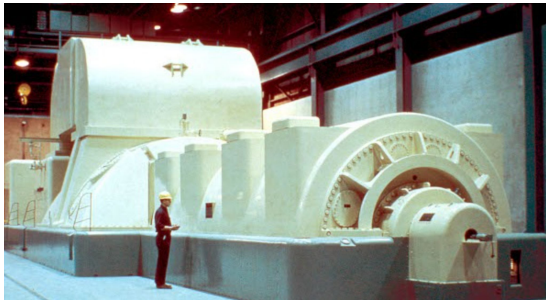
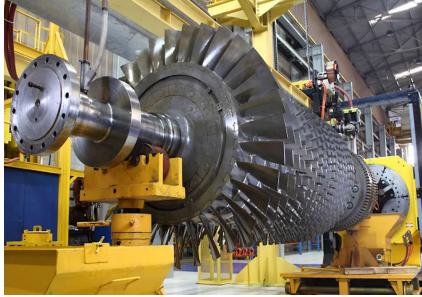
Negative Sequence



Zero Sequence

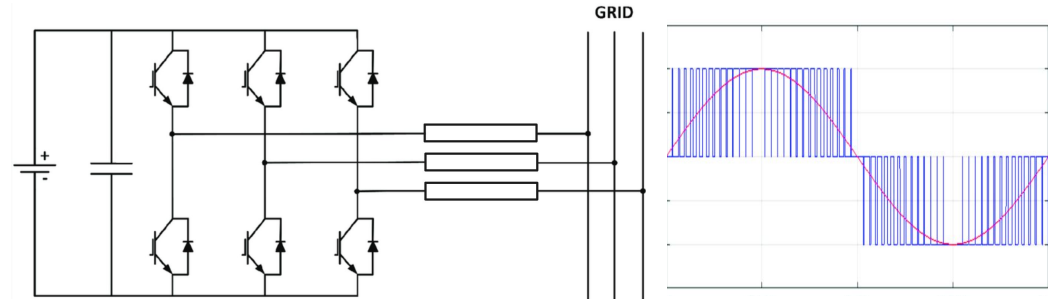
# Generators

## Rotating Machines



Dynamics: 
$$\frac{2H_i}{\omega_s} \frac{d^2\theta_i}{dt^2} = P_m - P_e$$

## Inverters

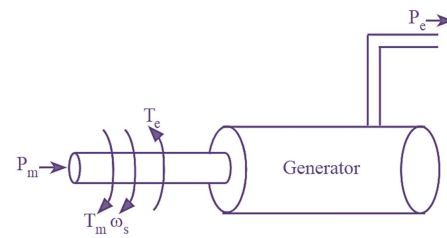


Dynamics: [insert control law here]



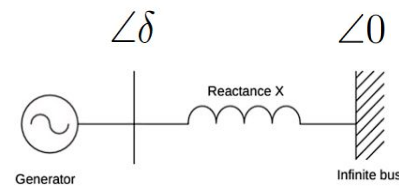
# The swing equation

The swing equation is the basic dynamic model for a single rotating machine



Swing equation for a single machine attached to an infinite bus:

$$\frac{2H_i}{\omega_s} \frac{d^2 \delta_i}{dt^2} = \frac{2H_i}{\omega_s} \frac{d\omega_i}{dt} = P_m - P_e$$

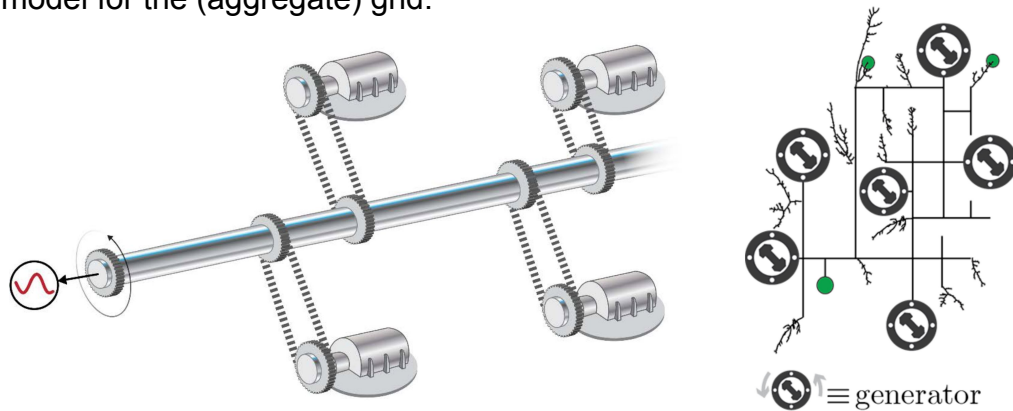


The swing equation also serves as a simple dynamic model for the (aggregate) grid:

$$\frac{2H_{\text{sys}}}{\omega_s} \frac{d\omega}{dt} = P_{\text{gen}} - P_{\text{load}}$$

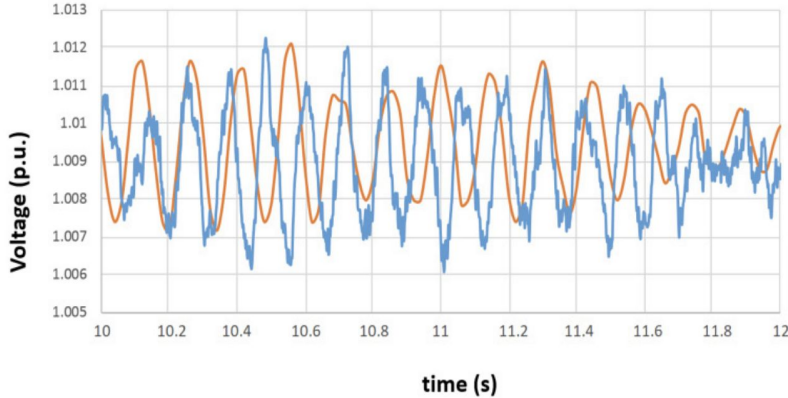
$H$ : inertia constant

$\omega_s$ : steady state system frequency

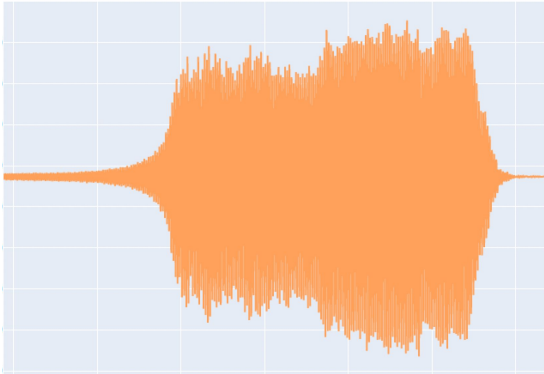


When power systems are unstable, they oscillate

Oscillations observed in southern Australia



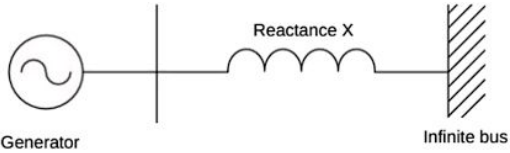
Oscillations observed in Northern Scotland



... or fail catastrophically.

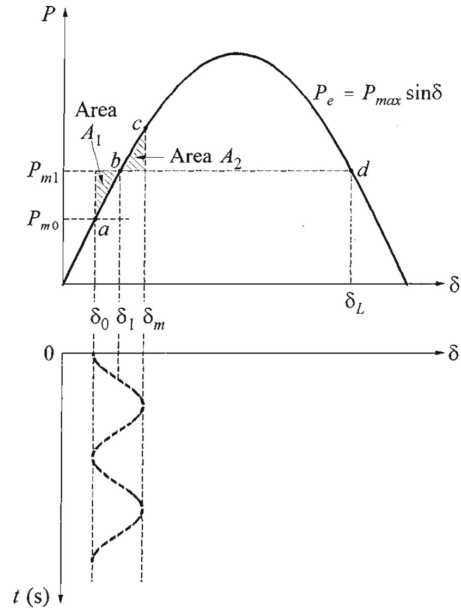
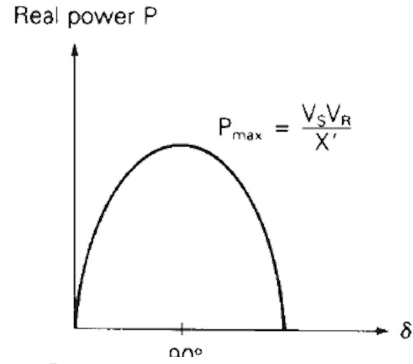


# The Equal Area Criterion

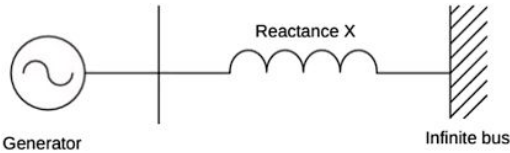


- Single machine
- No network resistance

$$\int_{\delta_0}^{\delta_1} (P_m - P_e) d\delta = \int_{\delta_1}^{\delta_m} (P_m - P_e) d\delta$$

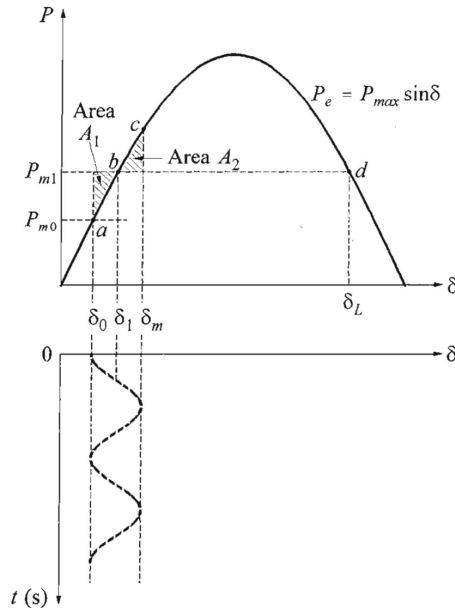
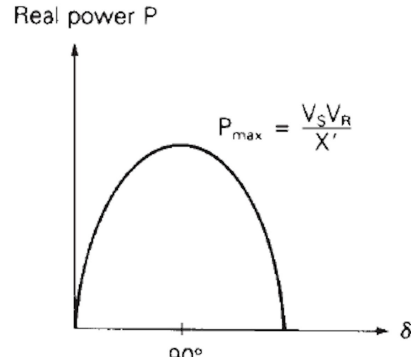


# The Equal Area Criterion



- Single machine
- No network resistance

$$\int_{\delta_0}^{\delta_1} (P_m - P_e) d\delta = \int_{\delta_1}^{\delta_m} (P_m - P_e) d\delta$$



...is a special case of Lyapunov stability

$$V(\delta, \tilde{\omega}) = \underbrace{\frac{1}{2} M \tilde{\omega}^2}_{\text{kinetic energy}} - \underbrace{P_m(\delta - \delta^s) - P_e^{\max}(\cos \delta - \cos \delta^s)}_{\text{potential energy}}$$

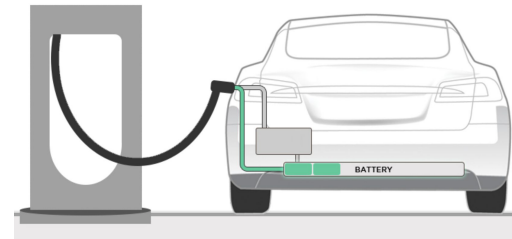
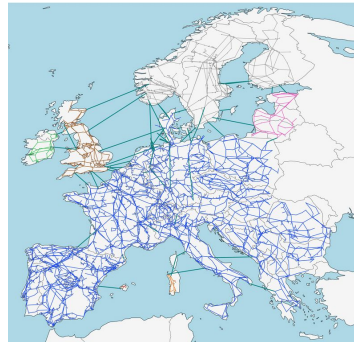
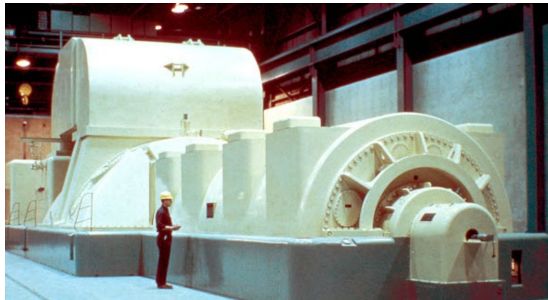
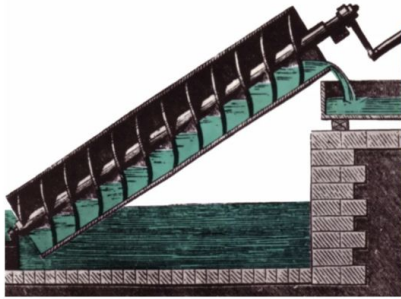
**ENERGY FUNCTION  
ANALYSIS FOR  
POWER SYSTEM  
STABILITY**

M. A. Pai



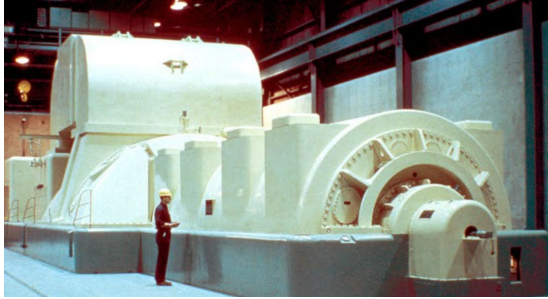
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# Power balance

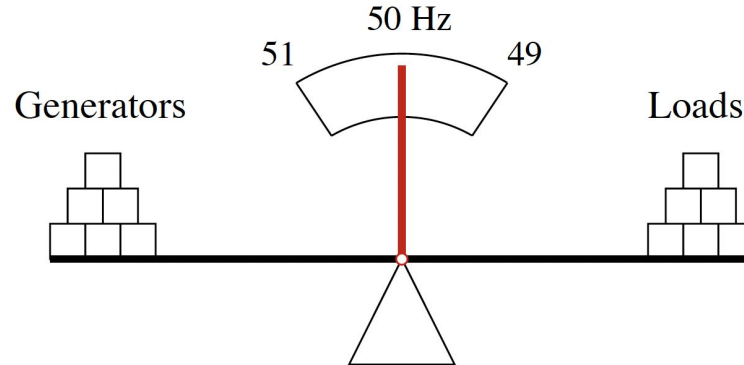


# Power balance

Power in  
(generated)

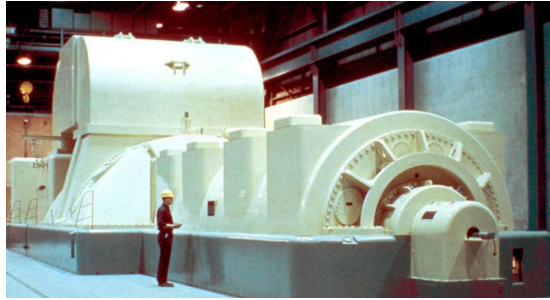


Power out  
(consumed)



# Power balance

Power in  
(generated)

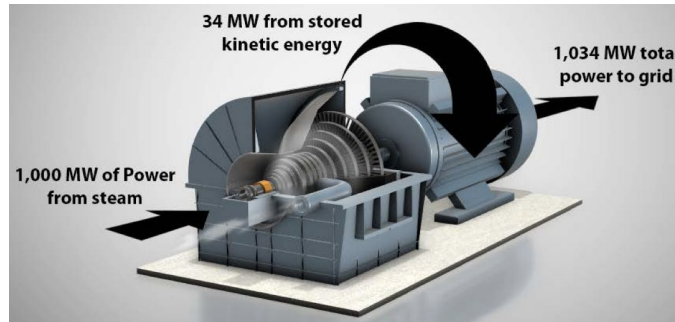


Power out  
(consumed)



What happens when power in  $\neq$  power out?

- The rotating machine inertia provides the power imbalance
- The rotating machine slows down (the frequency decreases)

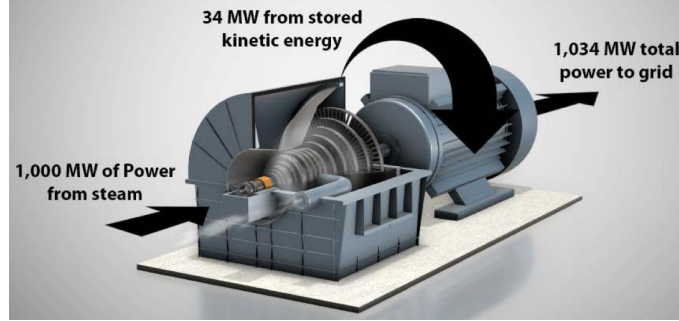




# Inertia

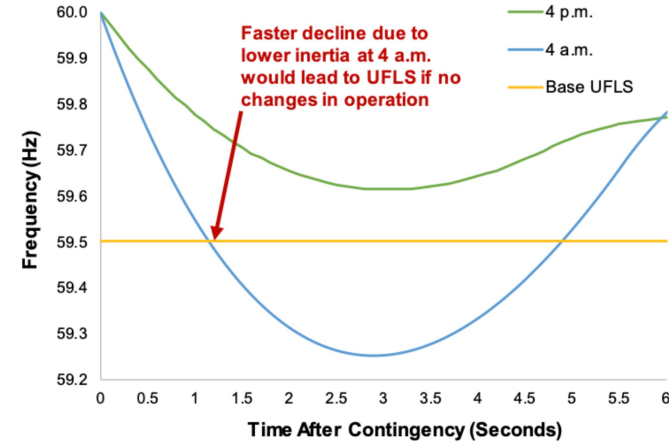
What happens when power in  $\neq$  power out?

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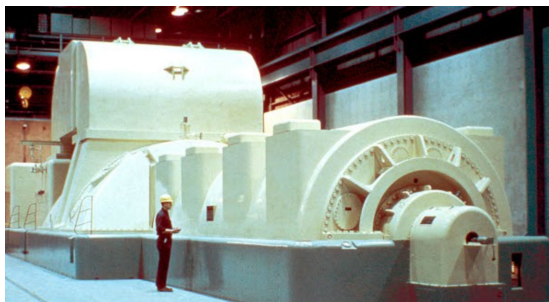
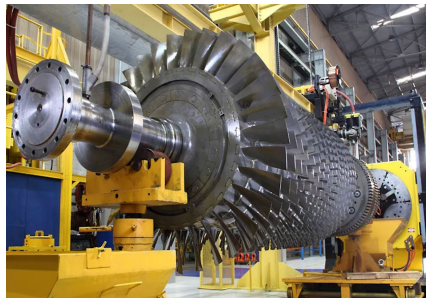
If there is less inertia, the frequency changes faster.

This is undesirable because the grid frequency has tight constraints.



# Generators

## Rotating Machines

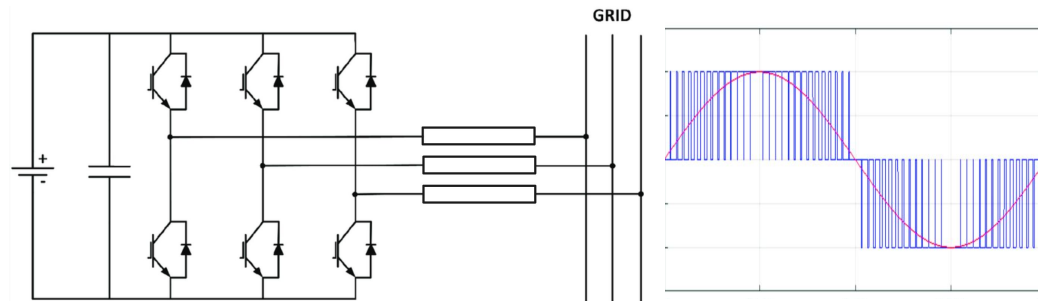


Dynamics: 
$$\frac{2H_i}{\omega_s} \frac{d^2\theta_i}{dt^2} = P_m - P_e$$

Inertia



## Inverters

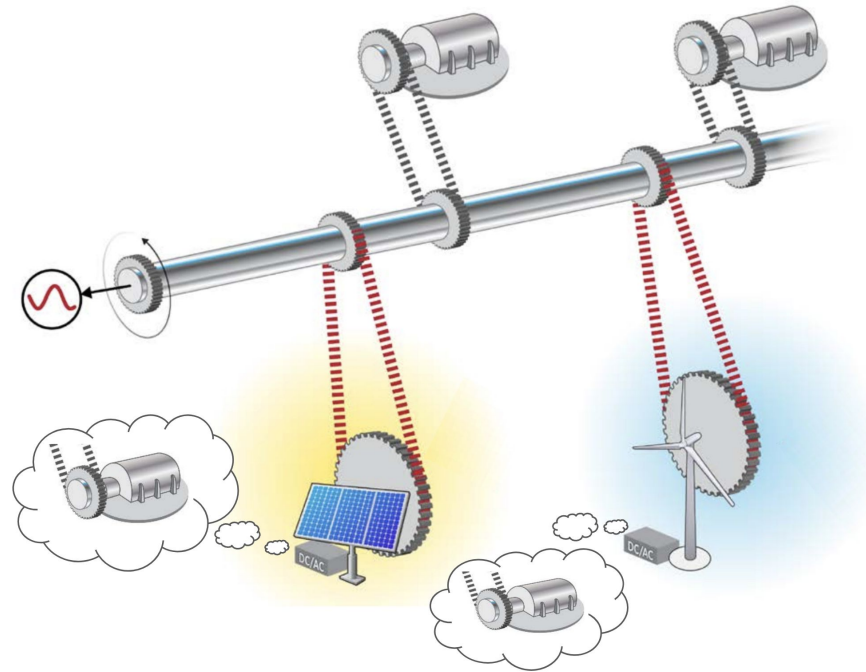


Dynamics: [insert control law here]



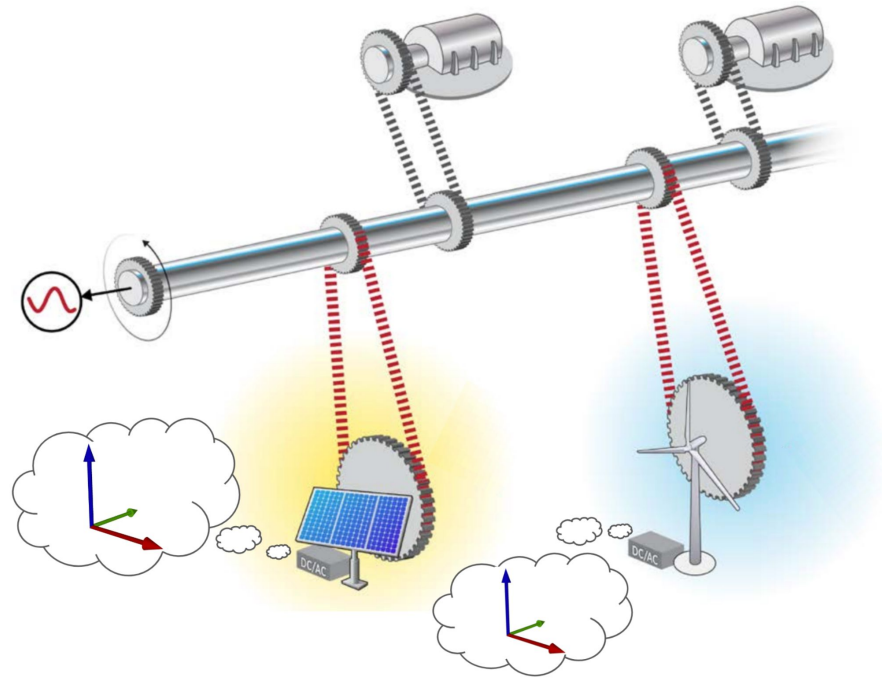
# Inverter control

Inverters can be controlled to mimic inertia of rotating machines



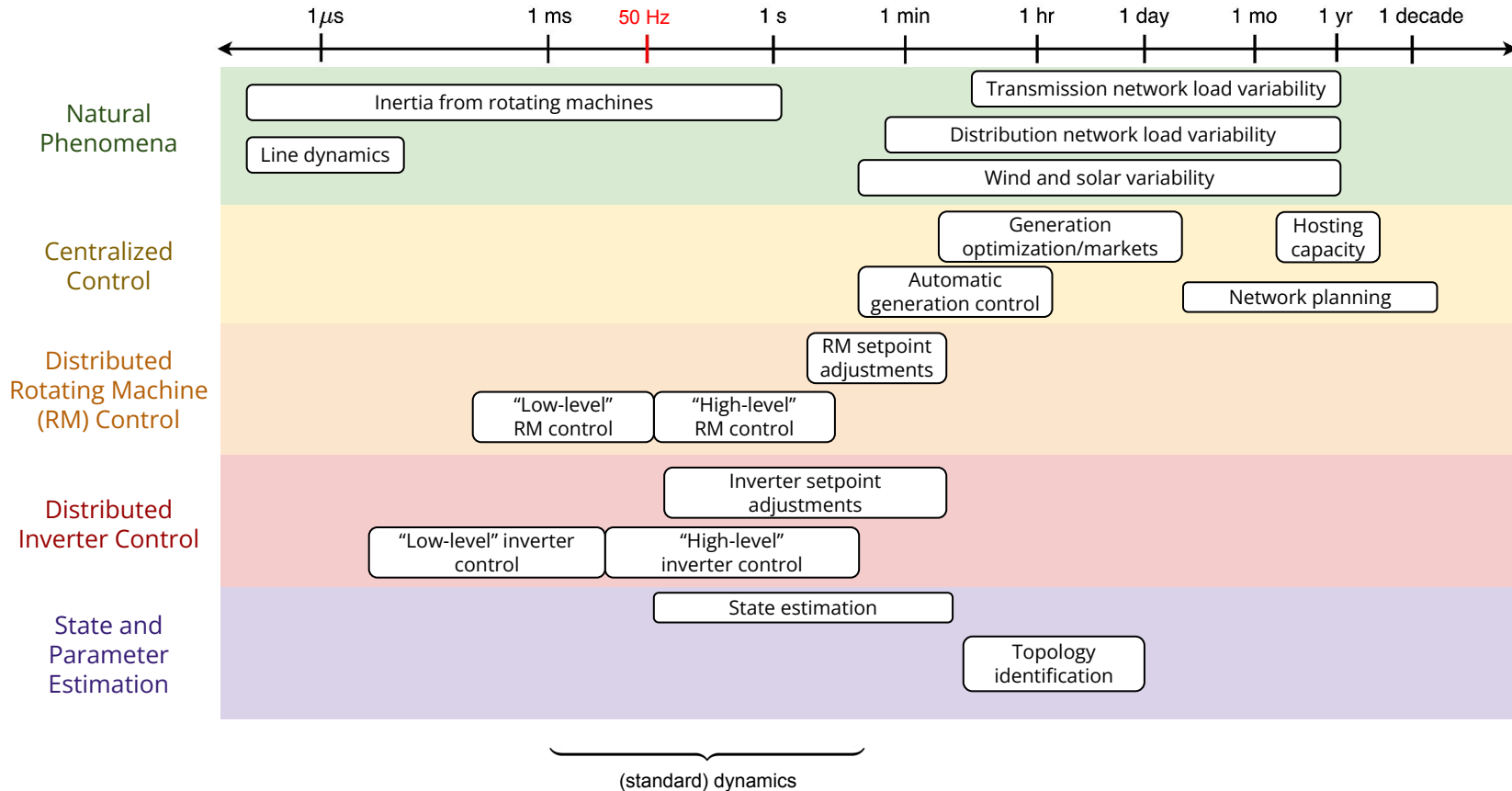
# Inverter control

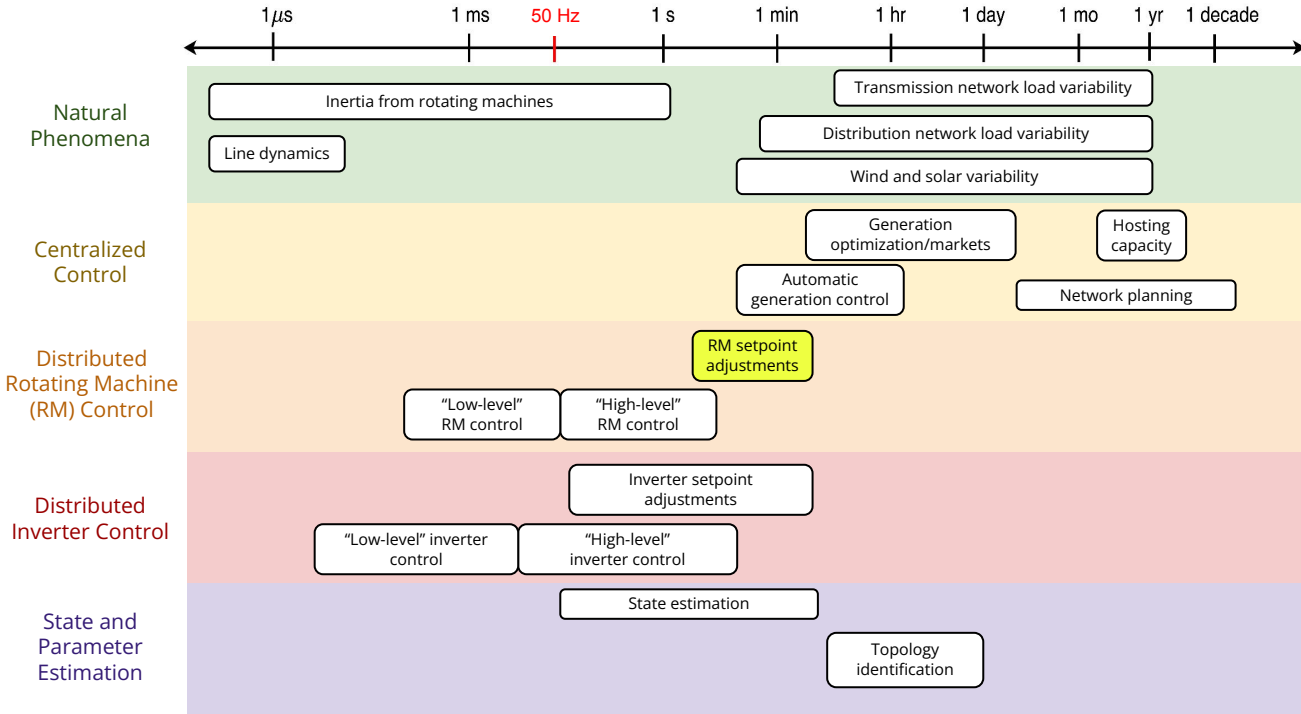
Inverters can be controlled to mimic inertia of rotating machines



... or to be even more useful for grid stability.

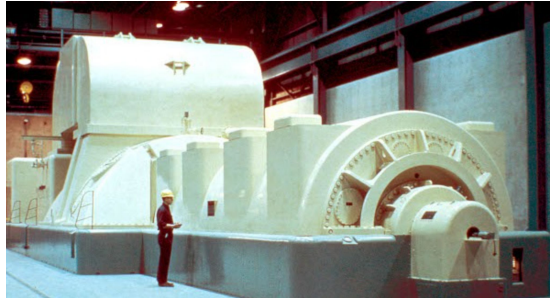
# Power Systems Timescales





# Power balance

Power in  
(generated)

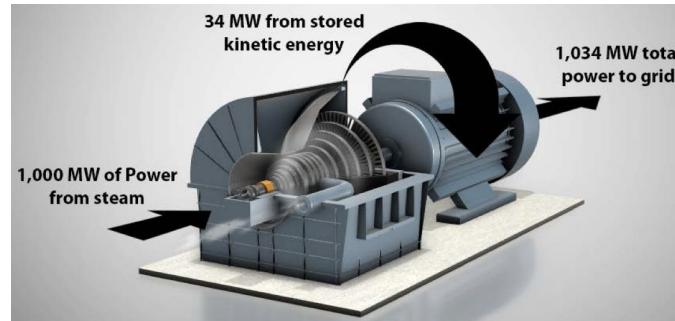


Power out  
(consumed)



What happens when power in  $\neq$  power out?

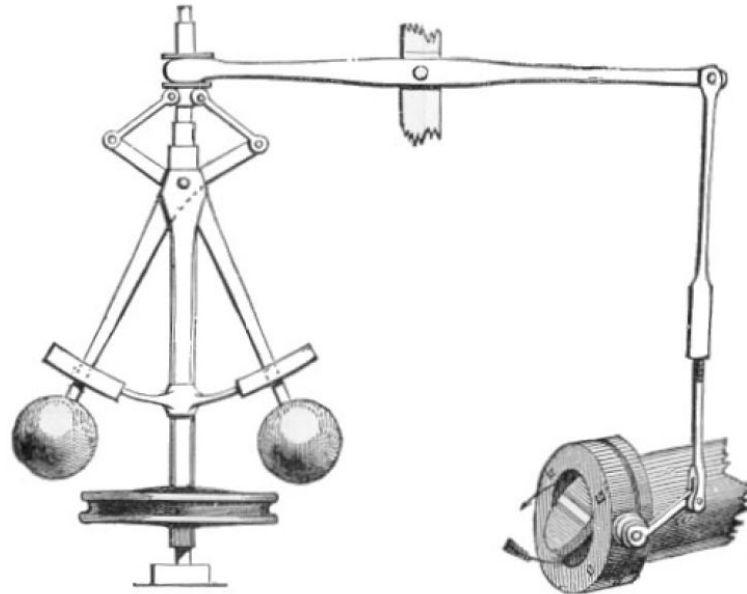
- The rotating machine inertia provides the power imbalance
- The rotating machine slows down (the frequency decreases)



What prevents the machine from slowing down too much?

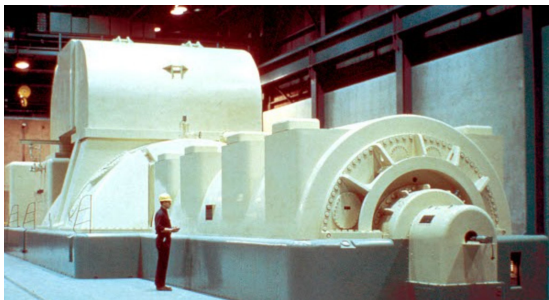
How did they coordinate many generators before computers?

# Droop control





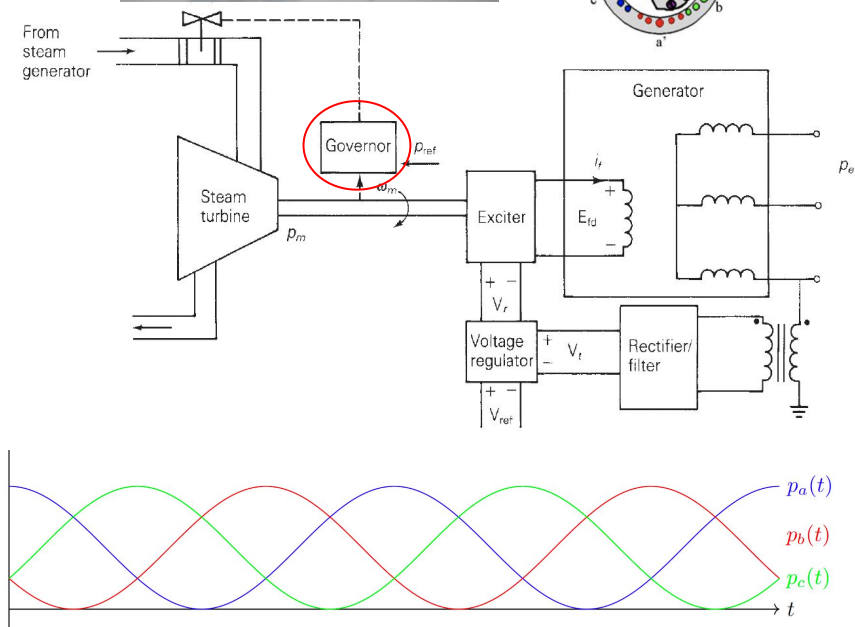
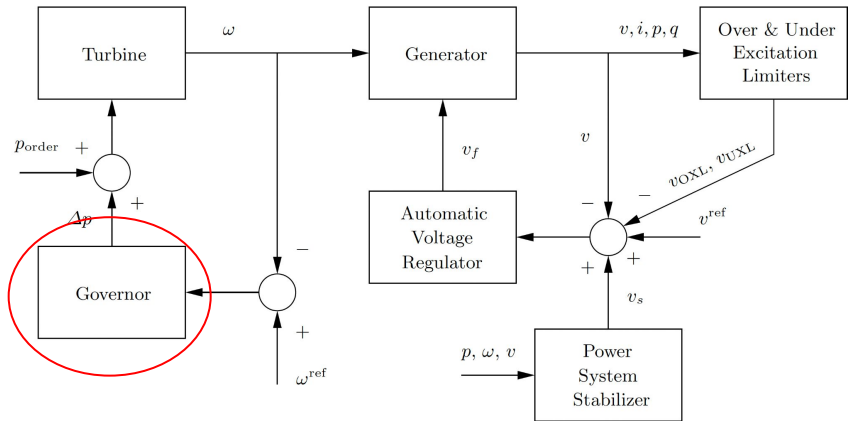
# Rotating machines

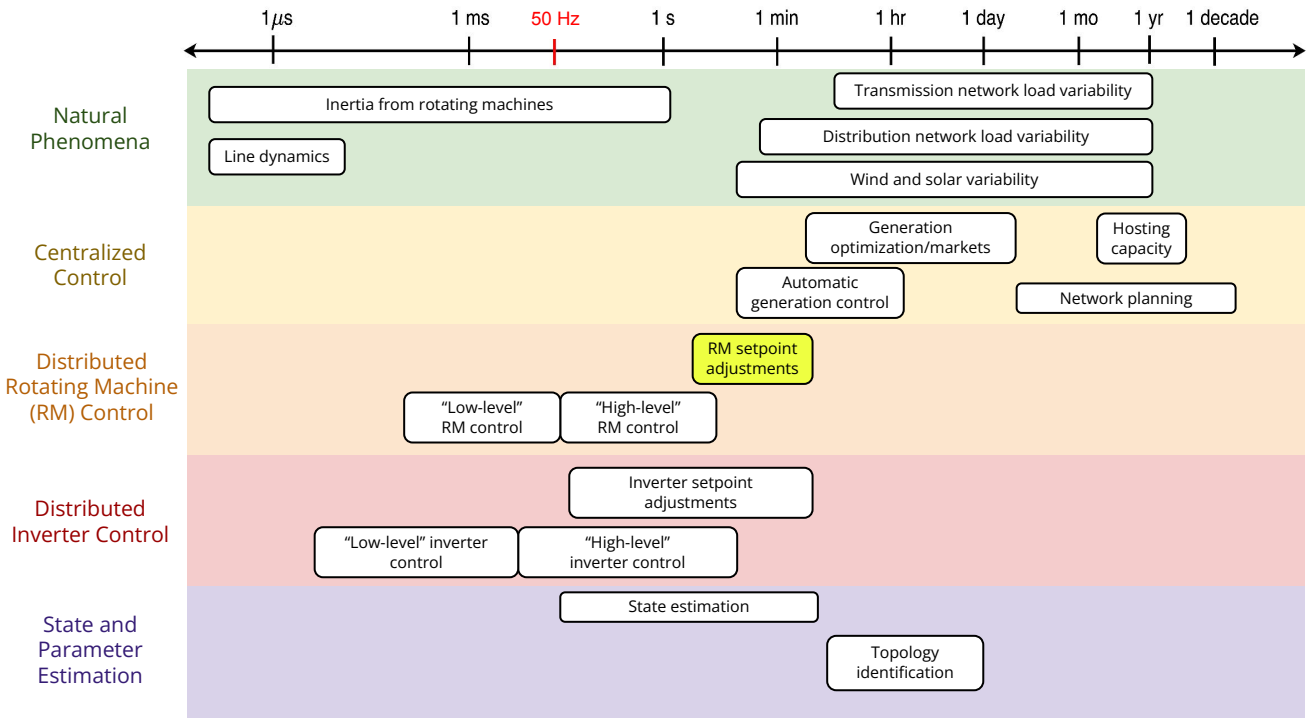


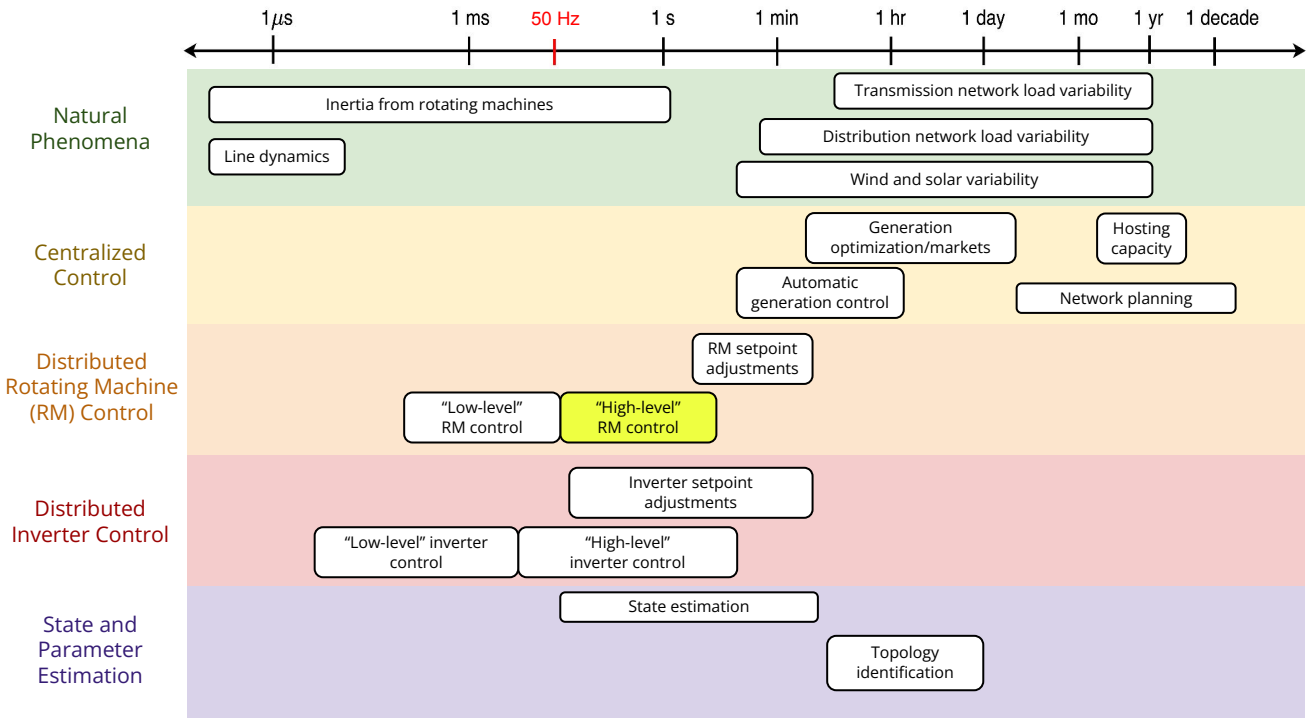
Schematic:



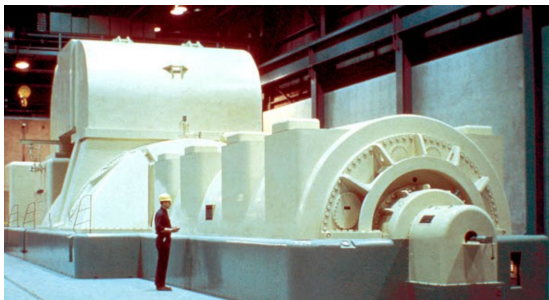
Block diagram:



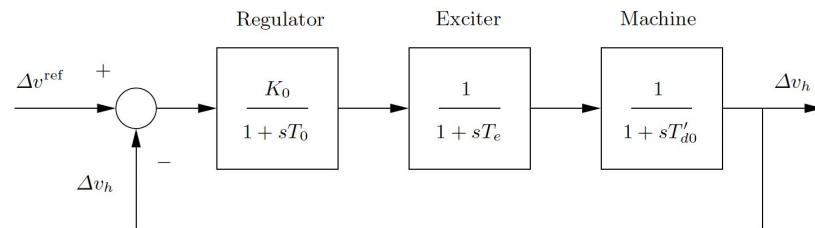




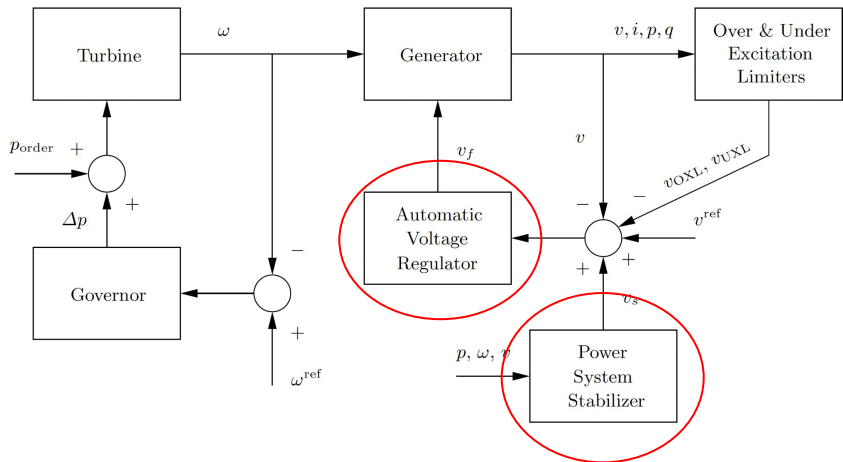
# Basic rotating machine control



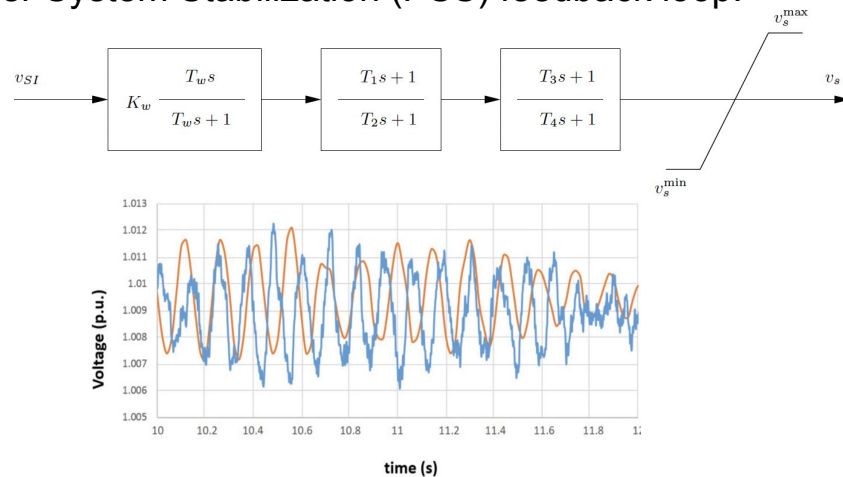
## Automatic Voltage Regulation (AVR) feedback loop:

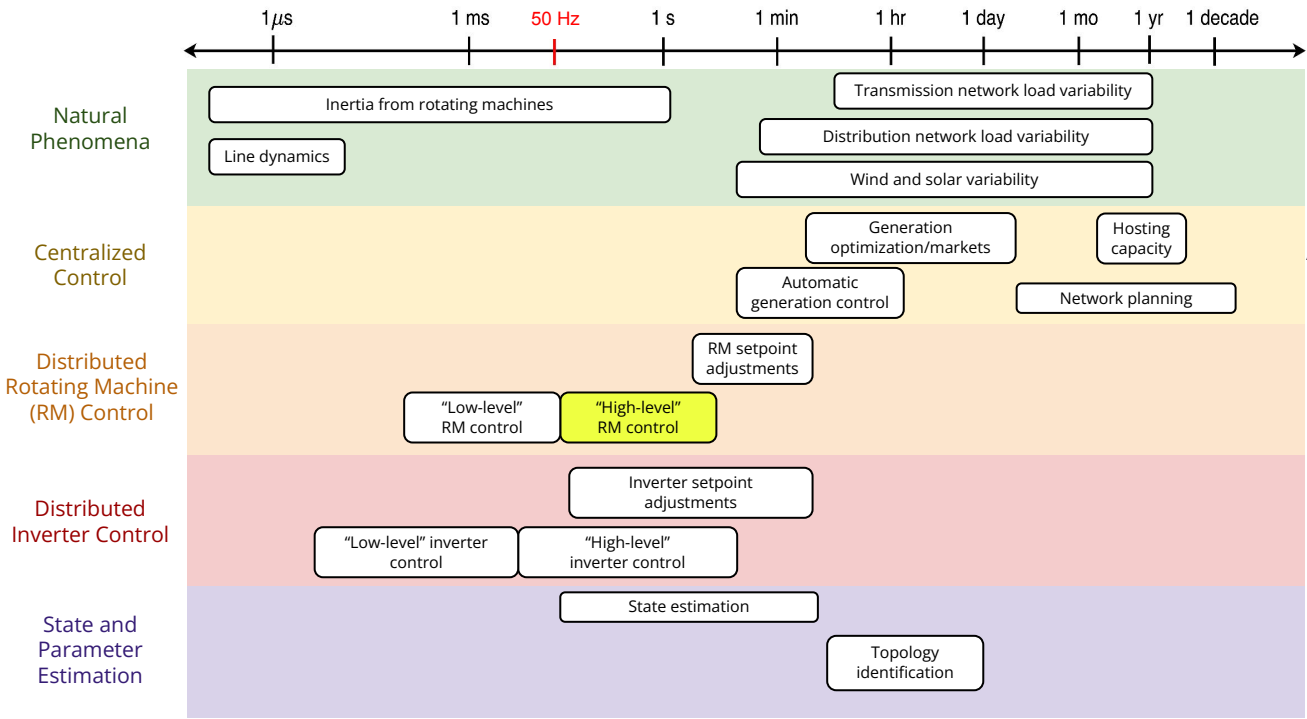


## Block diagram:

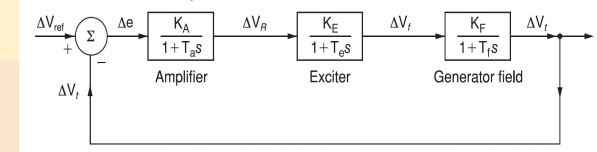


## Power System Stabilization (PSS) feedback loop:

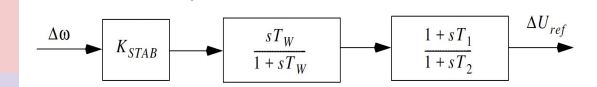


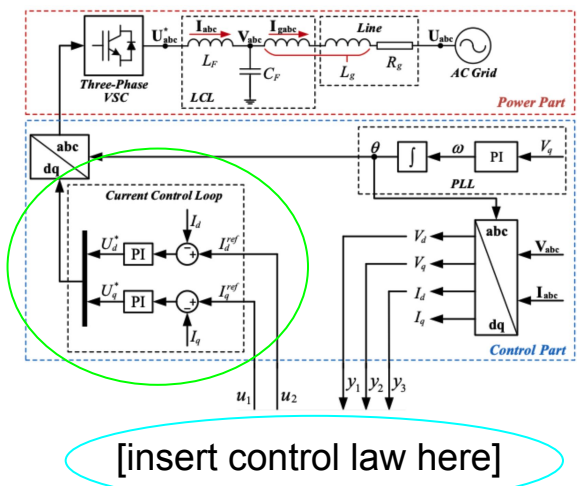
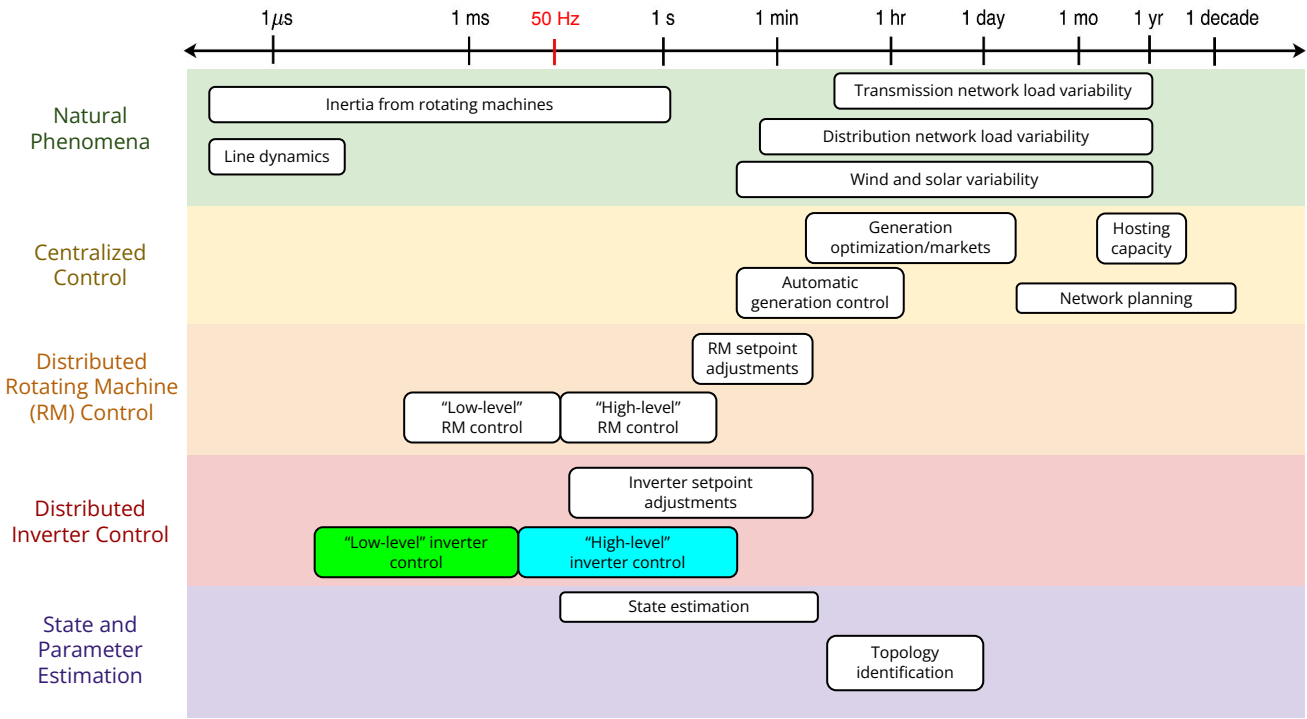


AVR feedback loop:



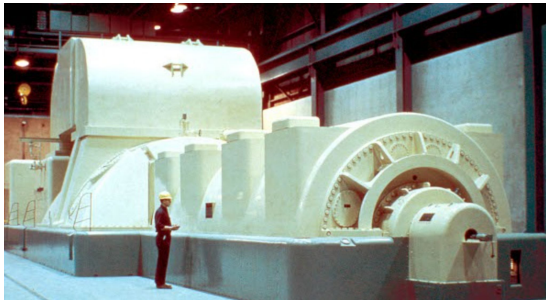
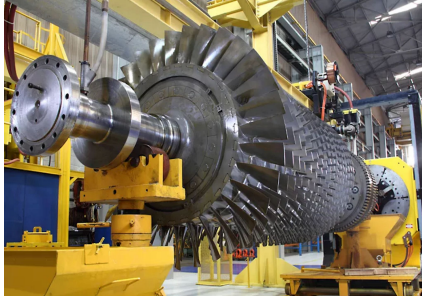
PSS feedback loop:





# Generators

## Rotating Machines

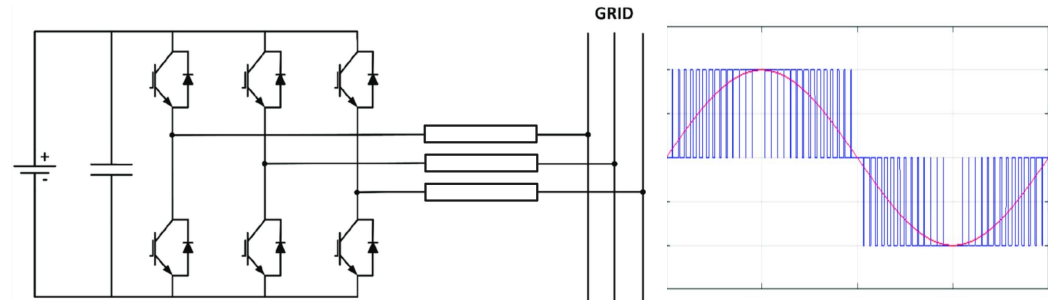


Dynamics: 
$$\frac{2H_i}{\omega_s} \frac{d^2\theta_i}{dt^2} = P_m - P_e$$

Inertia



## Inverters



Dynamics: [insert control law here]



# Grid Following (GFL) vs. Grid Forming (GFM) inverter control

## Definition 1:

GFL: behave as current sources

GFM: behave as voltage sources

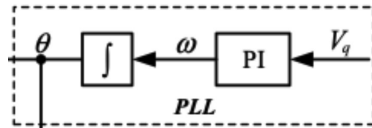
## Definition 2:

GFL: could not support a microgrid on its own

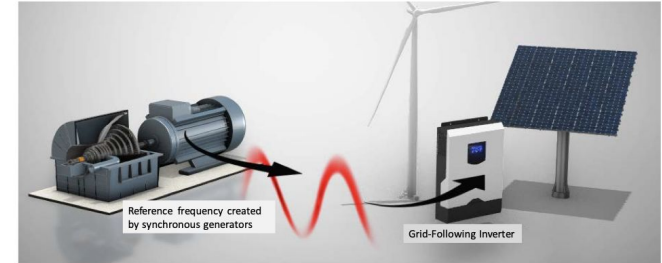
GFM: could support a microgrid on its own

## Definition 3:

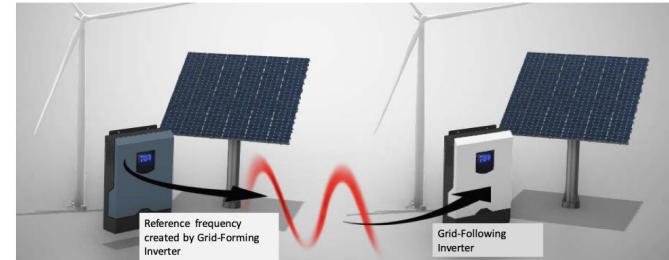
GFL: requires a frequency measurement, e.g. from a Phase-Locked Loop



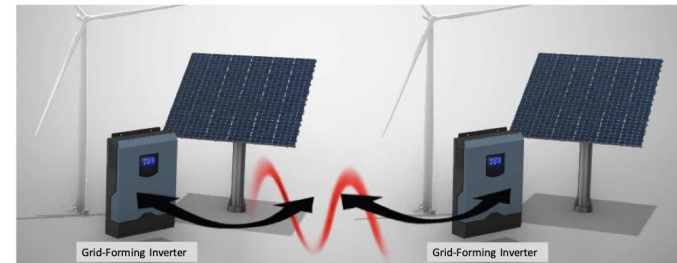
GFM: does not require an explicit frequency measurement.



a) Grid-following inverters in a synchronous grid



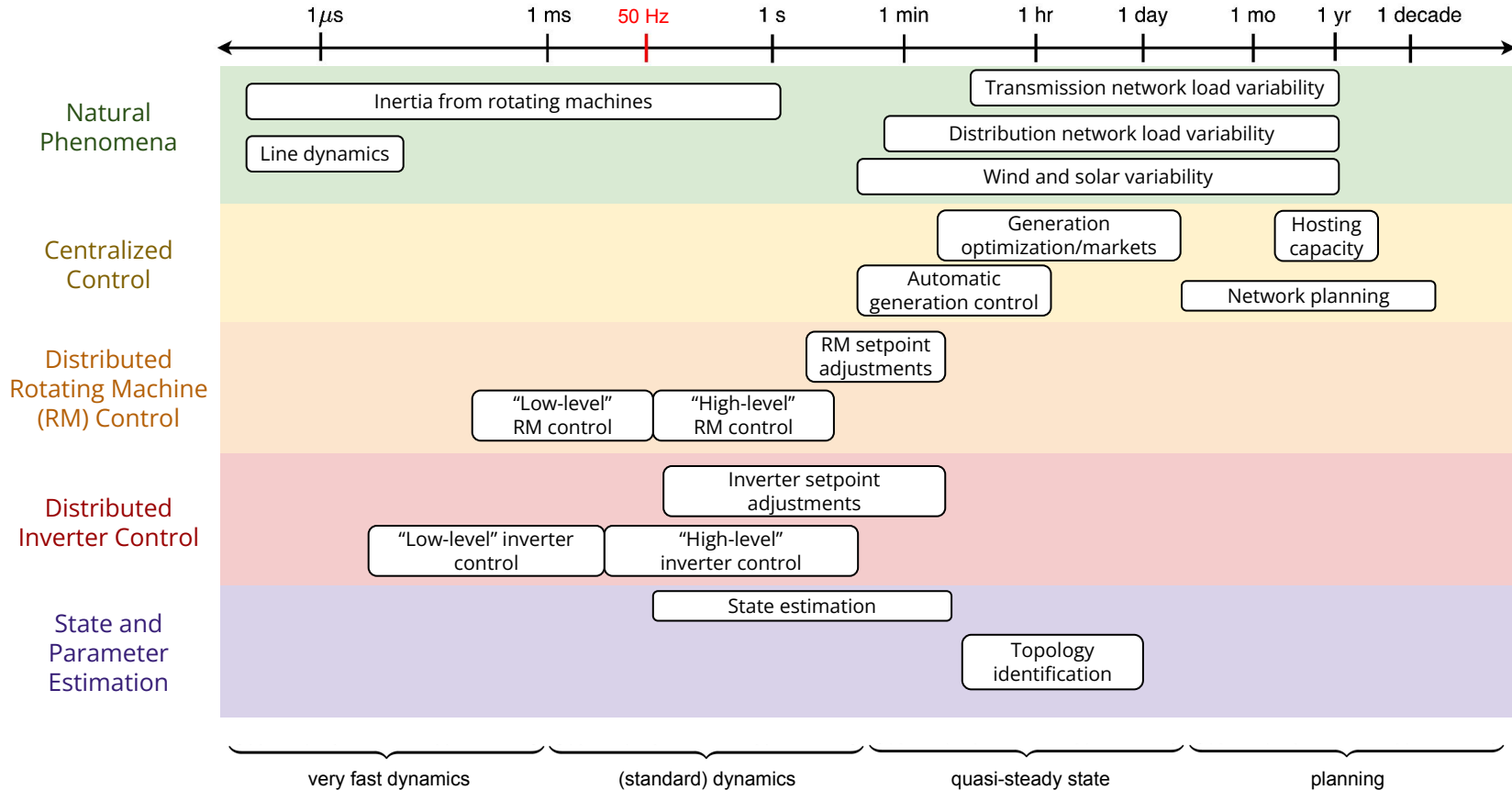
b) Grid-forming (and grid-following) inverters in VG-based microgrid



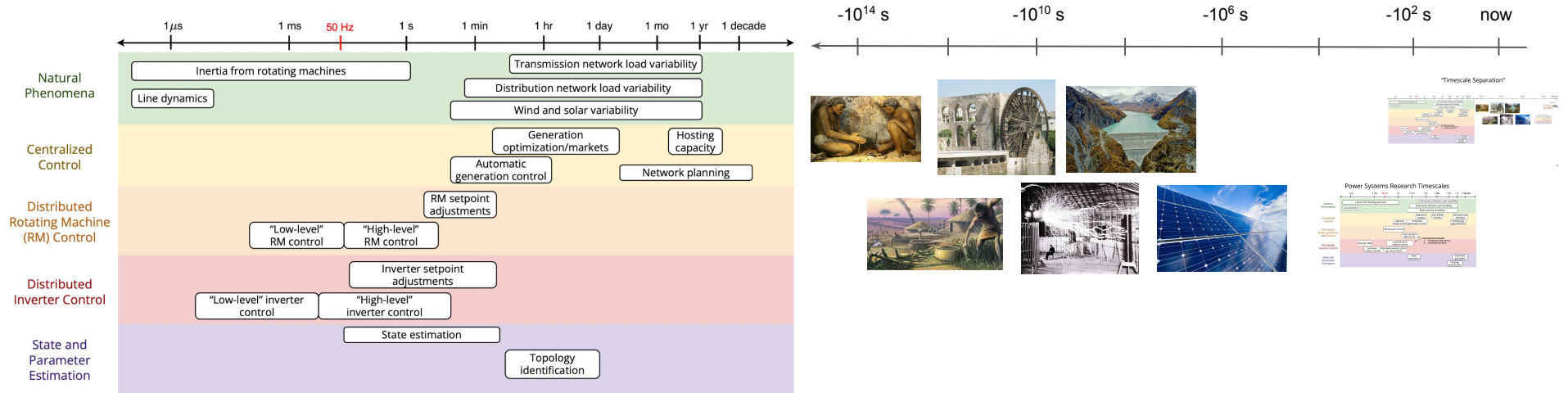
c) Grid-forming inverters in a large VG-based grid



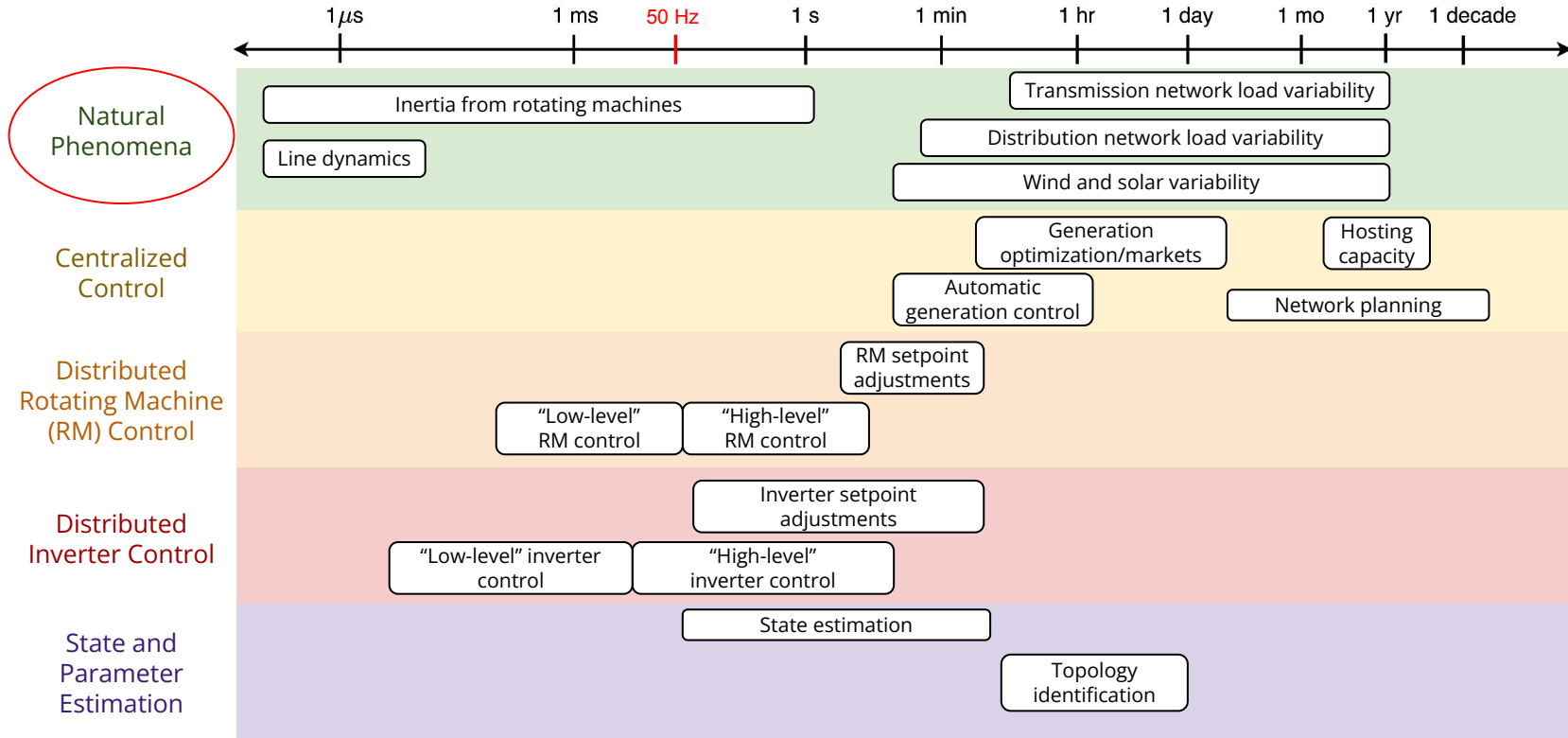
# Power Systems Timescales

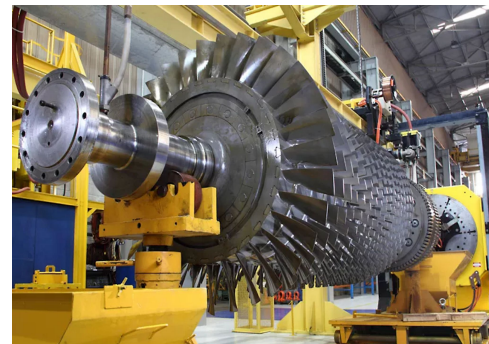
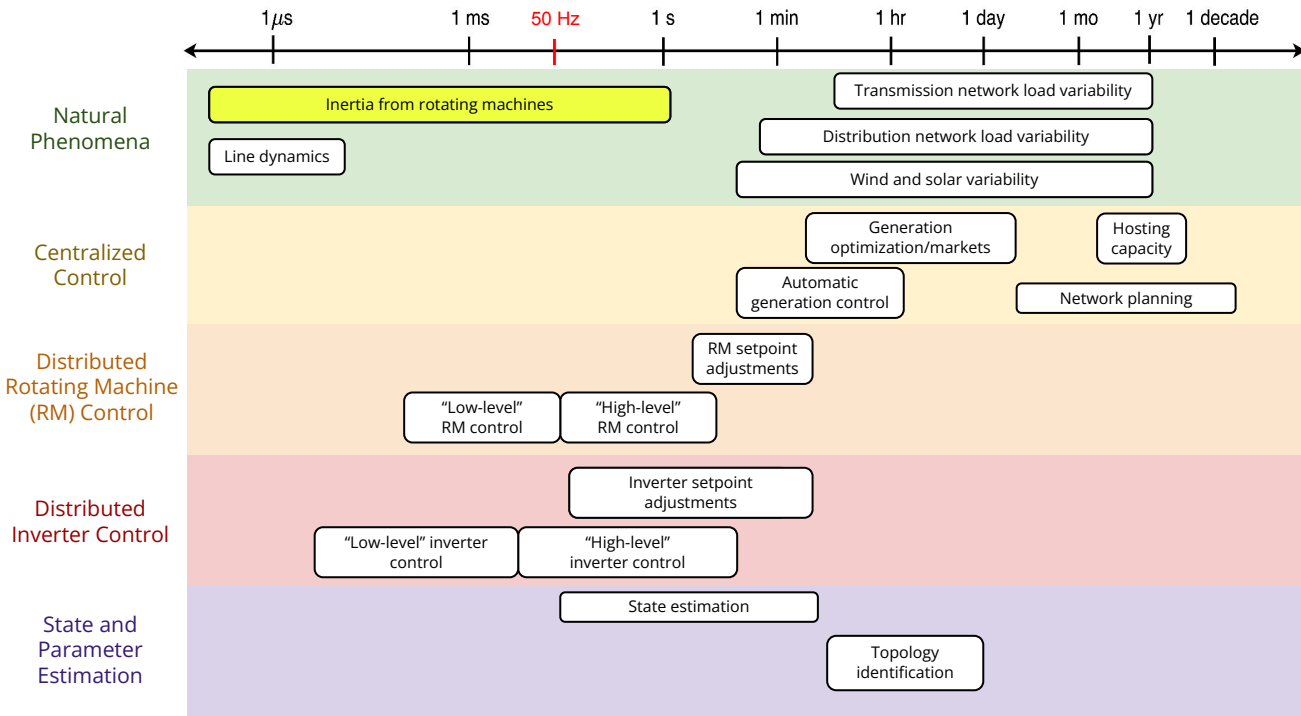


# Timescale Separation

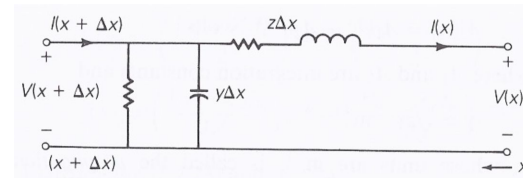
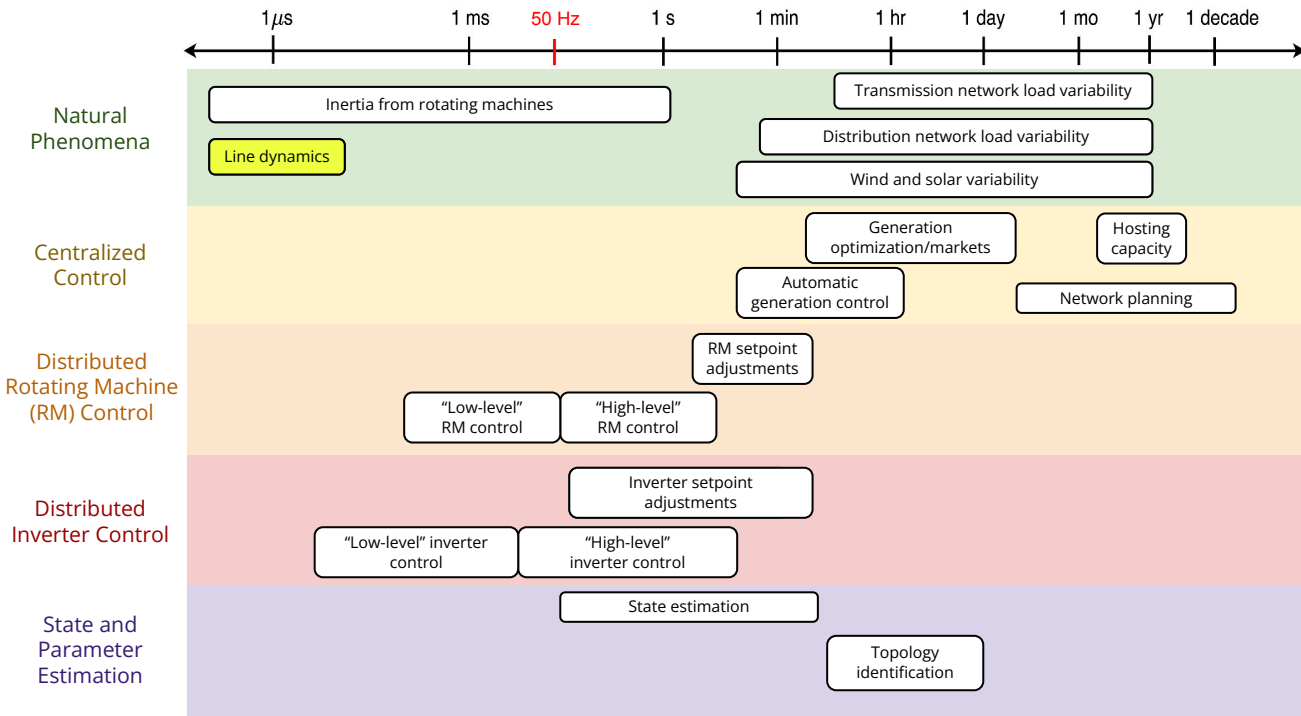


# Power Systems Timescales





$$\frac{2H_i}{\omega_s} \frac{d^2\theta_i}{dt^2} = P_m - P_e$$



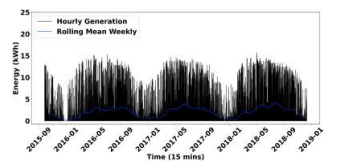
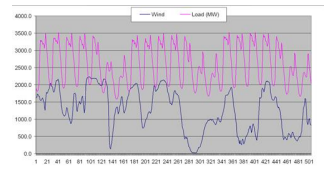
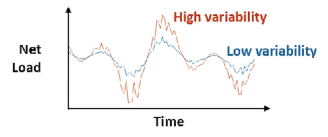
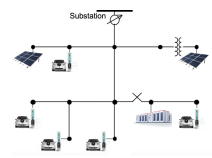
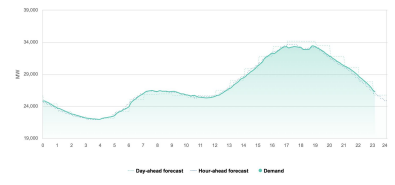
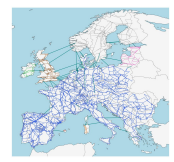
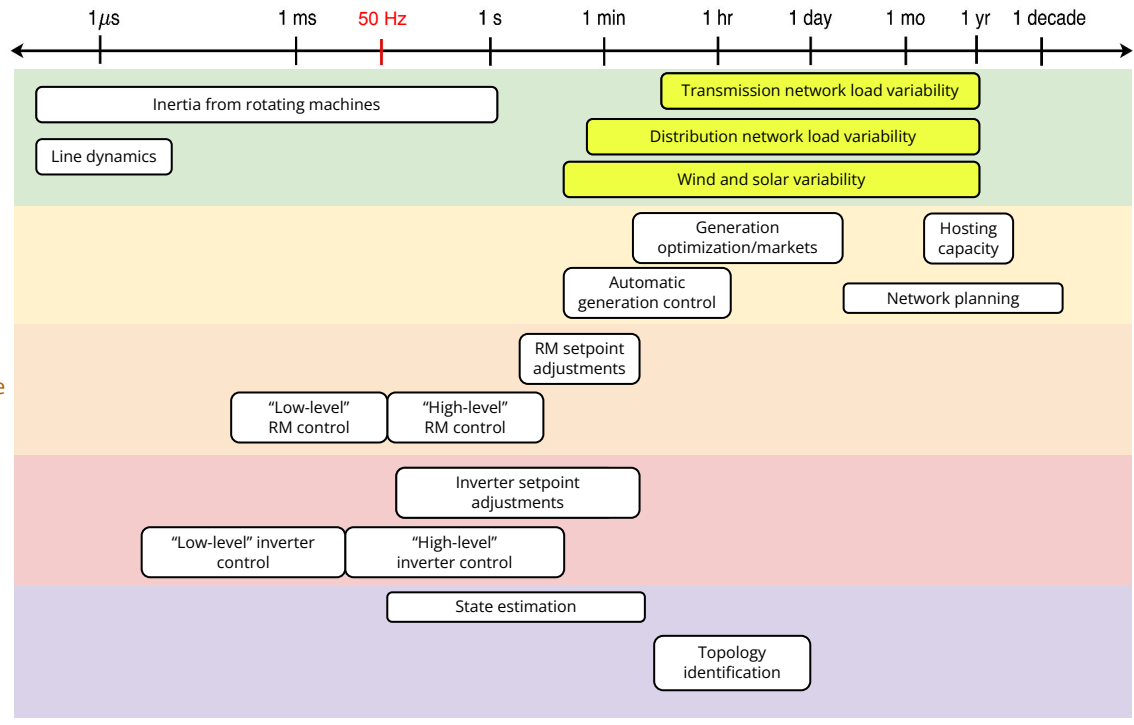
$$v = f\lambda$$

$$\lambda = 3000 \text{ km}$$

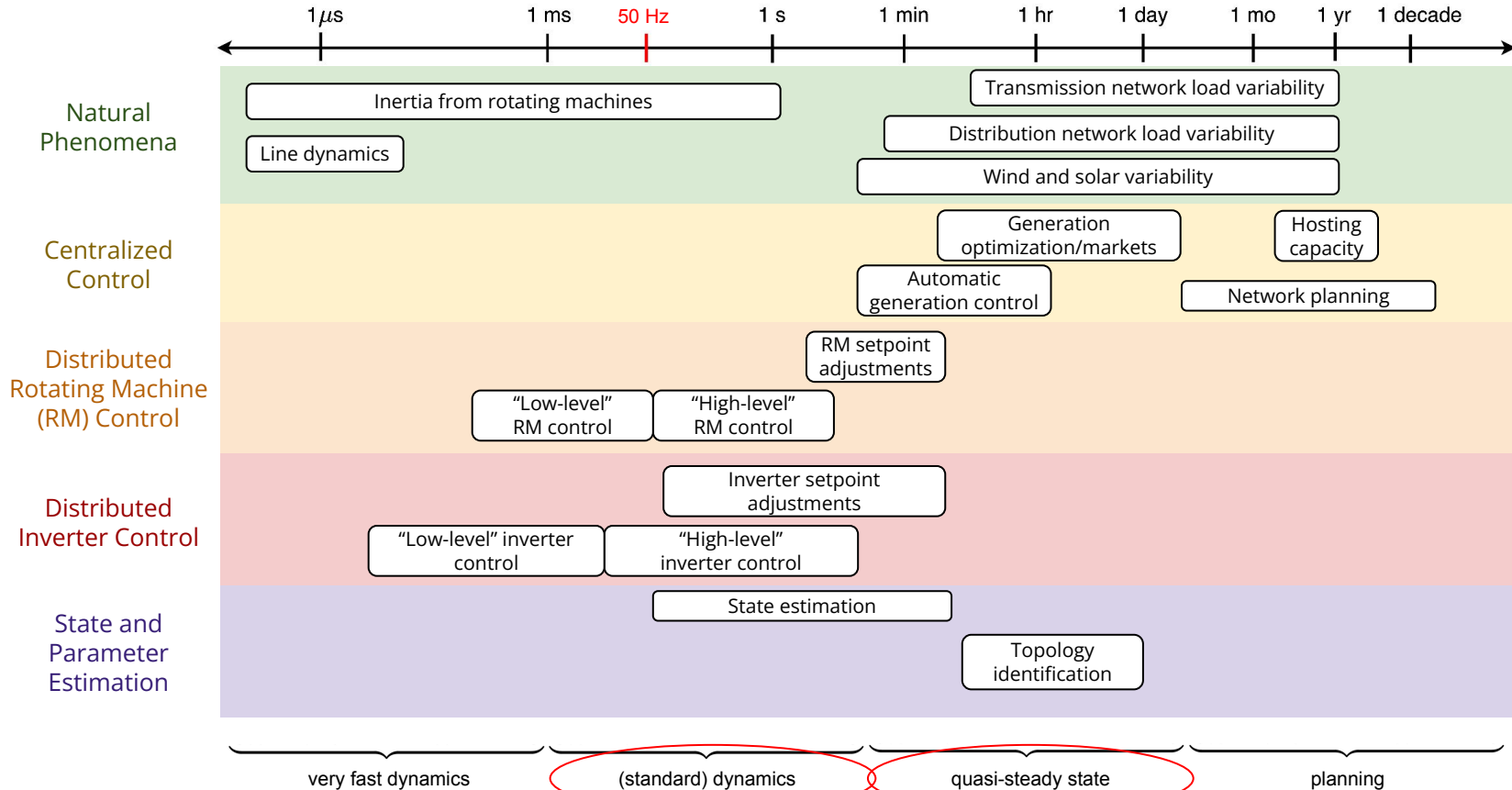
$$f = 50 \text{ Hz}$$

$$v = 150 \text{ km/s}$$

$$= .0005c$$



# Power Systems Timescales

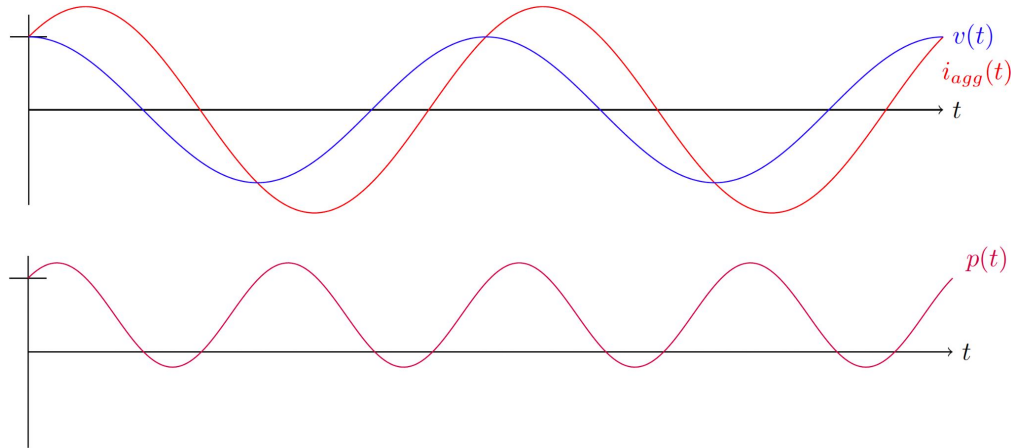


# Steady state

The swing equation is an example of a dynamic model of the grid  $\implies \frac{2H_i}{\omega_s} \frac{d^2 \delta_i}{dt^2} = \frac{2H_i}{\omega_s} \frac{d\omega_i}{dt} = P_m - P_e$

At steady state, the power in is equal to the power out at each bus and the frequency is constant  $\implies \frac{d\omega_i}{dt} = 0$

The instantaneous power on any line is given by multiplying the voltage by the current  $\implies p(t) = v(t)i(t)$





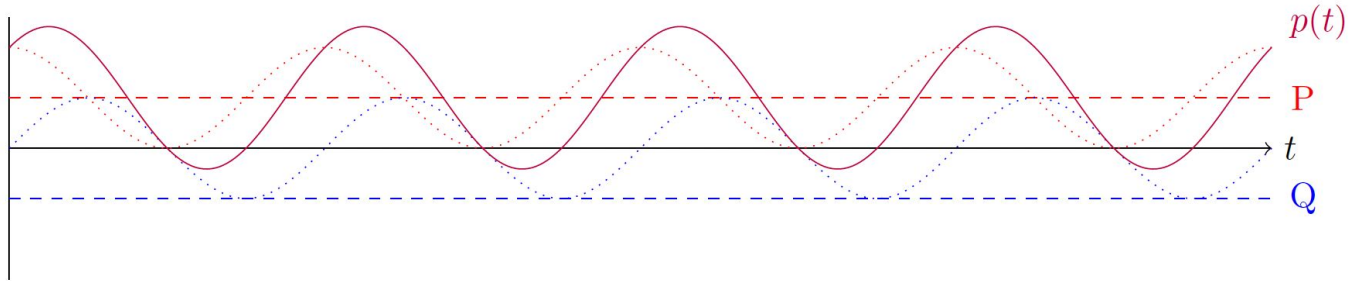
# Real and reactive power

Real power is the average power that is delivered over a (steady state) AC cycle

Reactive power is the magnitude of the power that oscillates back and forth each (steady state) AC cycle

Apparent power is the complex sum of real and reactive power  $\implies S = P + Qj$

Apparent power is computed using the voltage and current phasors  $\implies S = VI^*$



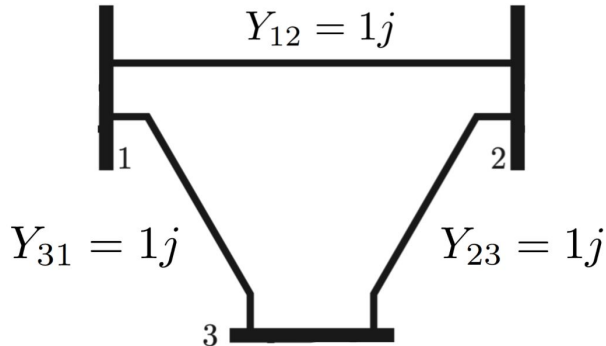
# Bus admittance matrix

The bus admittance matrix allows us to write Ohm's law for vectors of currents and voltages

$$I = \overset{\downarrow}{Y}V$$

$$Y(i, k) = \begin{cases} y_i + \sum_{l \neq i} y_{il}, & \text{if } i = k \\ -y_{ik}, & \text{if } i \neq k \end{cases}$$

Example:



$$Y = \begin{bmatrix} 2j & -1j & -1j \\ -1j & 2j & -1j \\ -1j & -1j & 2j \end{bmatrix}$$

## Power flow equations

$$\begin{aligned} S &= \text{diag}(V)I^* \\ &= \text{diag}(V)(YV)^* \end{aligned}$$

# Power flow equations

In terms of complex voltage (for the whole network):

$$\begin{aligned} S &= \text{diag}(V)I^* \\ &= \text{diag}(V)(YV)^* \end{aligned}$$

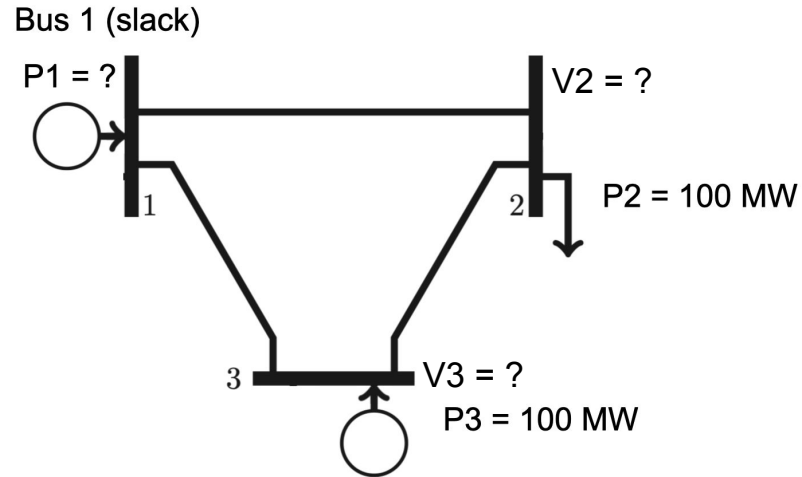
In terms of polar, real-valued voltage (for a single bus):

$$\begin{aligned} P_i &= P_{Gi} - P_{Di} = \sum_{k=1}^N |V_i||V_k|(G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik}) \\ Q_i &= Q_{Gi} - Q_{Di} = \sum_{k=1}^N |V_i||V_k|(G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik}) \end{aligned}$$

# Solving the power flow equations

The power flow equations are a system of nonlinear equations

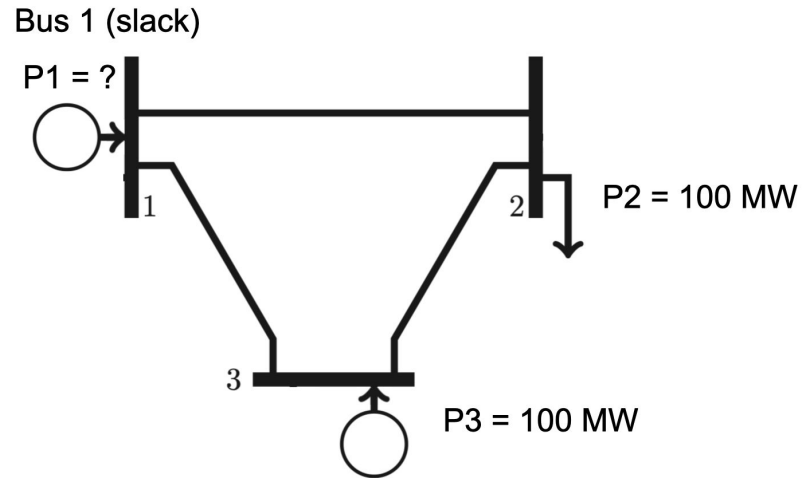
→ solved with iterative methods



# Slack bus

You don't know network losses ahead of time.

In order to have a viable set of power injections, the power injection at the “slack bus” is flexible.



# Optimal power flow

Finds the optimal steady state operating point

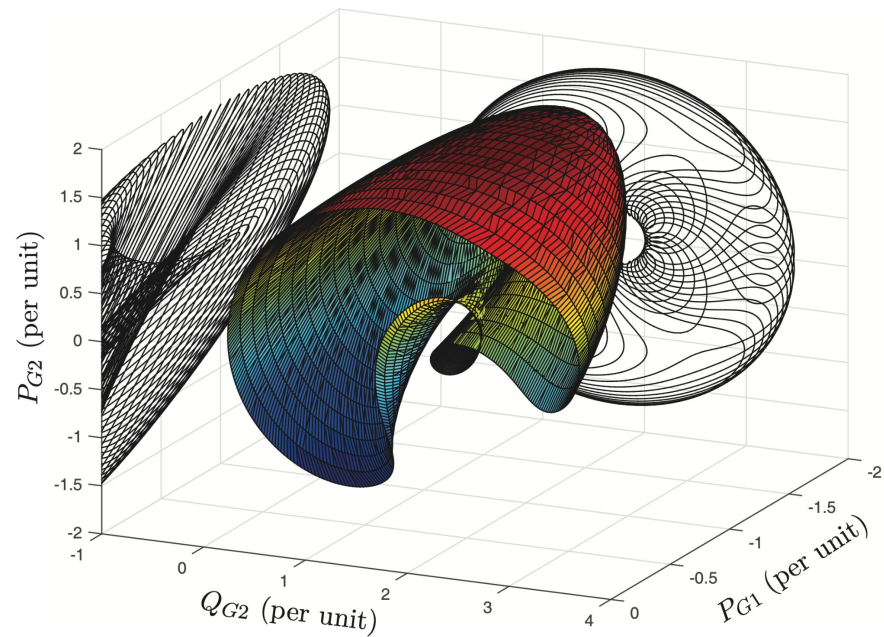
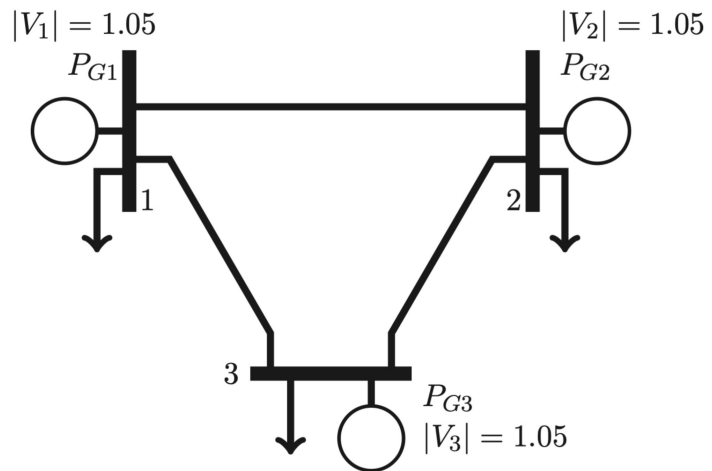
$$\min_{P, Q} f(P, Q)$$

s.t. Power flow equations,  
Network flow (thermal) constraints,  
Network bus (voltage) constraints,  
Individual generator constraints

What is the cost function?

What are the constraints?

# The power flow manifold





# Optimal power flow

Finds the optimal steady state operating point

$$\min_{P, Q} f(P, Q)$$

s.t. Power flow equations,  
Network flow (thermal) constraints,  
Network bus (voltage) constraints,  
Individual generator constraints

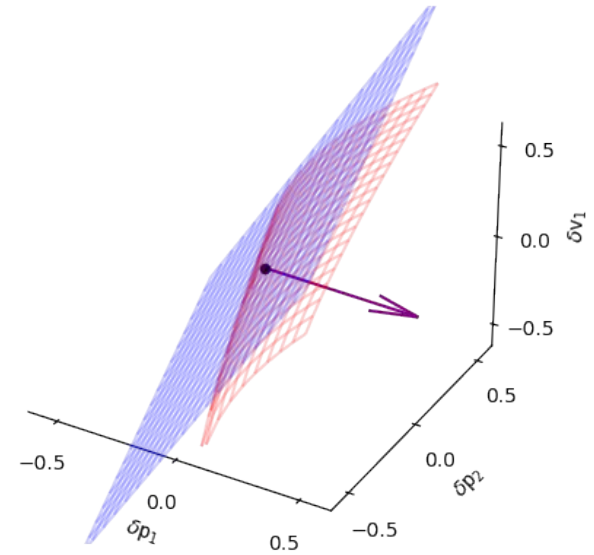
What is the cost function?

What are the constraints?

How do you handle the nonlinear power flow equations?

# Linearized (AC) power flow

Relates real and reactive power injections to voltage magnitudes and angles



# “DC” Power Flow

A special case of linearized AC Power Flow that relates (just) real power injections to voltage angles

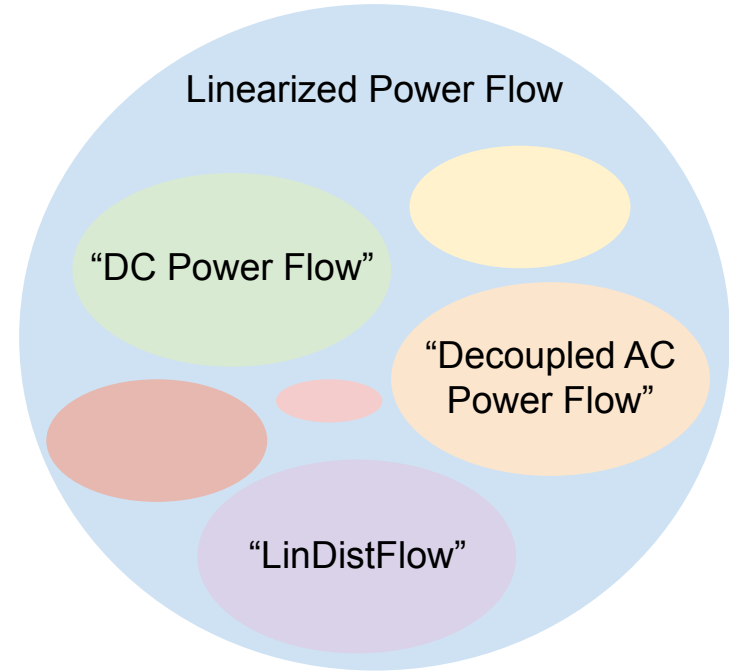
Assumptions:

- Linearized at the nominal ( $|V| = 1$  per unit) operating point
- The network has no resistance

DC Power Flow:

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_n \end{bmatrix} = \begin{bmatrix} c_1 & -b_{12} & -b_{13} & \cdots & -b_{1n} \\ -b_{21} & c_2 & -b_{23} & \cdots & -b_{2n} \\ -b_{31} & -b_{32} & c_3 & \cdots & -b_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -b_{n1} & -b_{n2} & -b_{n3} & \cdots & c_n \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \vdots \\ \theta_n \end{bmatrix}$$

$$c_i = \sum_{j \neq i} b_{ij}, \quad b_{ij} \text{ is the susceptance between node } i \text{ and node } j$$



## Dynamics, steady state, and quasi-steady state

Dynamics: The swing equations, etc.

$$\Longrightarrow \frac{2H_i}{\omega_s} \frac{d^2 \delta_i}{dt^2} = \frac{2H_i}{\omega_s} \frac{d\omega_i}{dt} = P_m - P_e$$

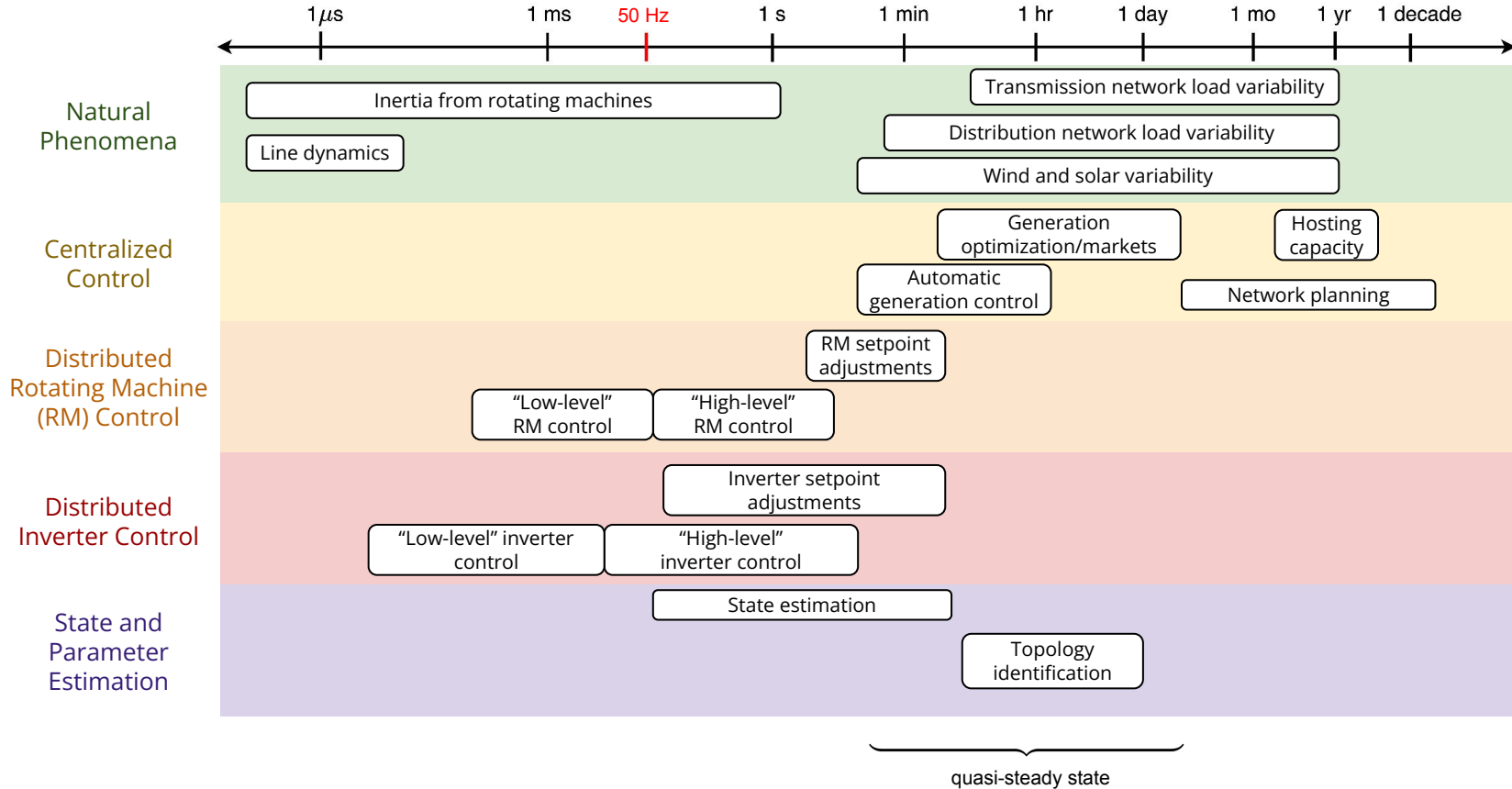
Steady state: The power flow equations

$$\Longrightarrow S = \text{diag}(V)(YV)^*$$

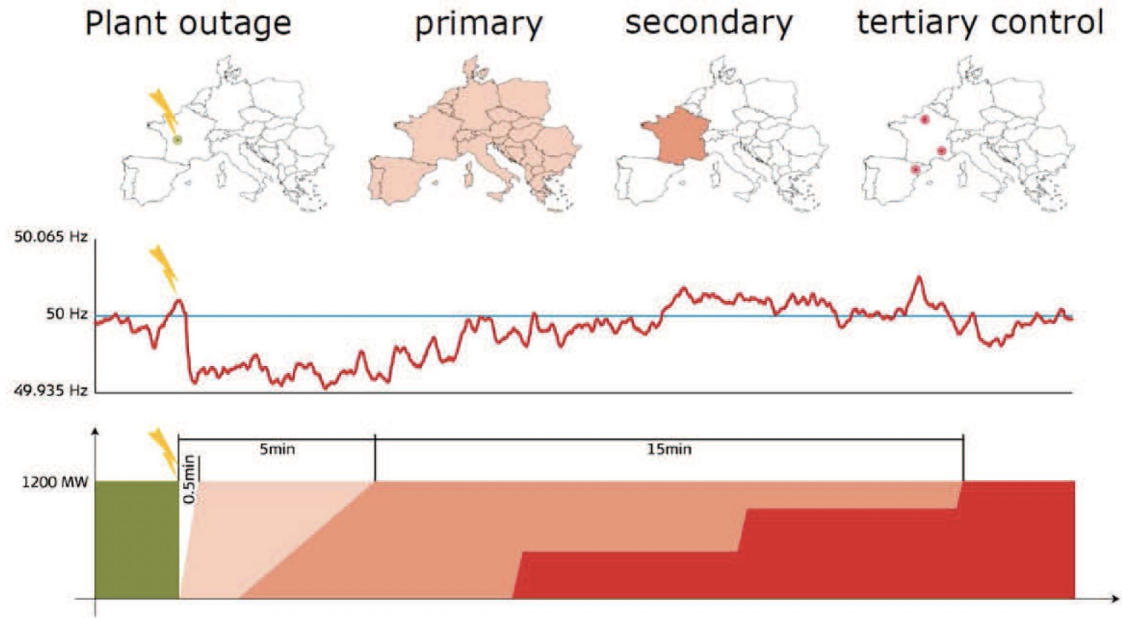
Quasi-steady state: Transitions from one steady state to another

$$\begin{aligned} \Longrightarrow S_t &= \text{diag}(V_t)(YV_t)^* \\ S_{t+1} &= \text{diag}(V_{t+1})(YV_{t+1})^* \end{aligned}$$

# Power Systems Timescales

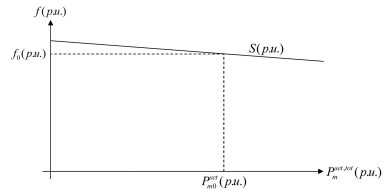
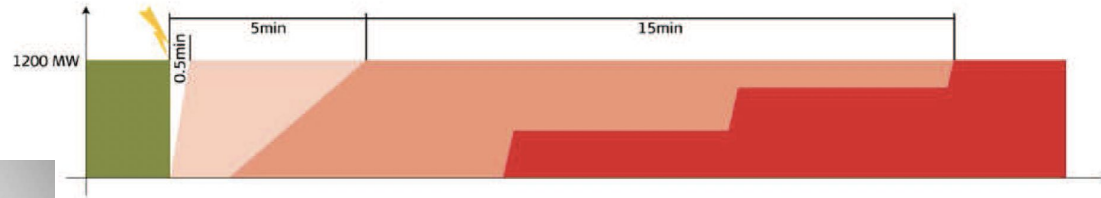
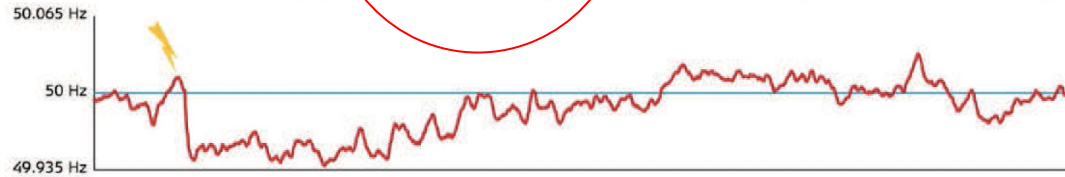


# Quasi steady state control

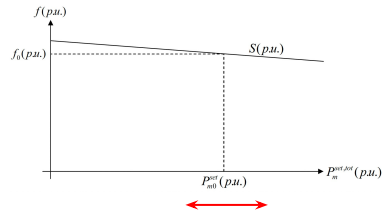
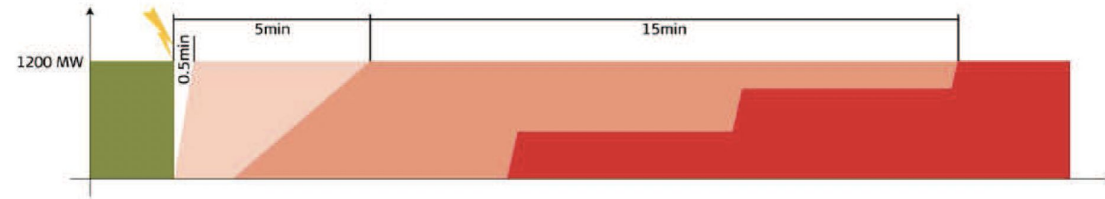
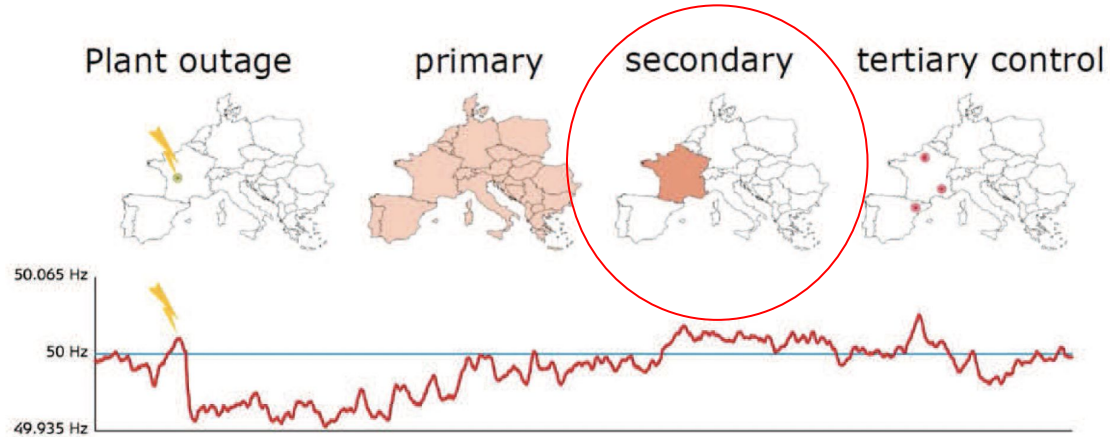


# Quasi steady state control

Plant outage      primary      secondary      tertiary control

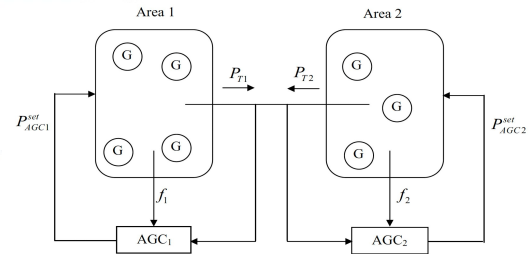


# Quasi steady state control



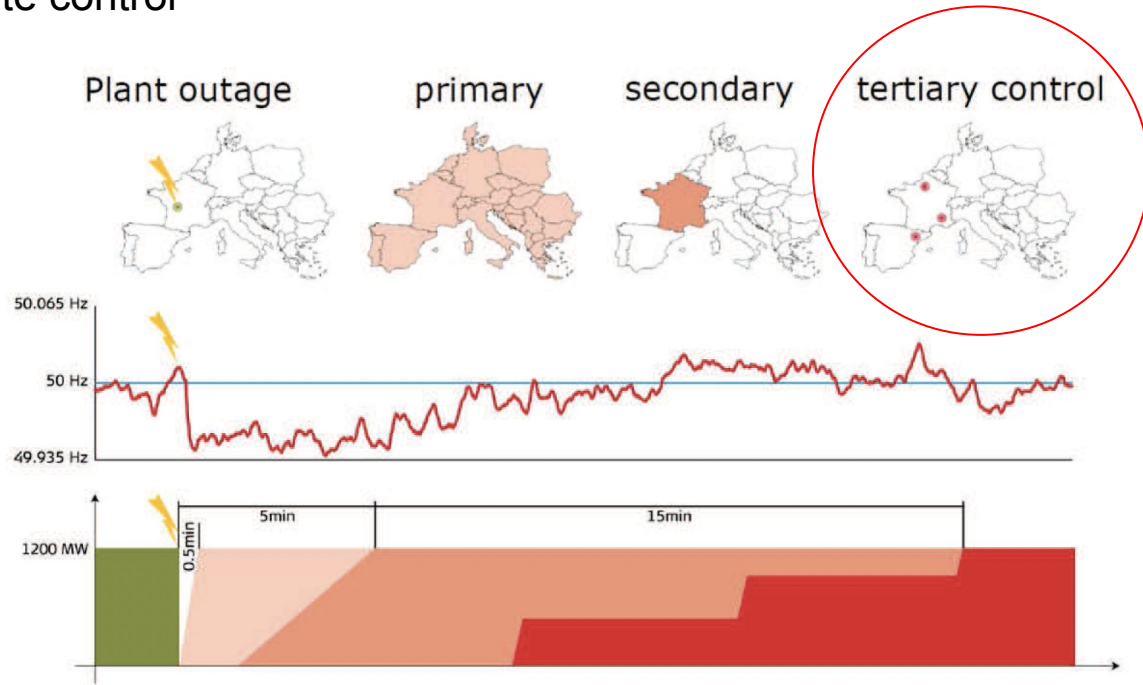
$$\Delta P_{AGCi} = -\left(C_{pi} + \frac{1}{sT_{Ni}}\right)\Delta e_i$$

$$ACE_i = \Delta P_{Ti} + B_i \Delta f \quad i = 1, 2, \dots, N$$





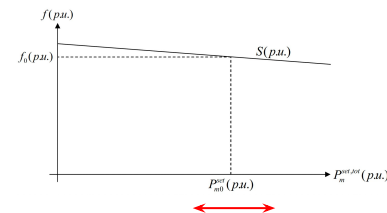
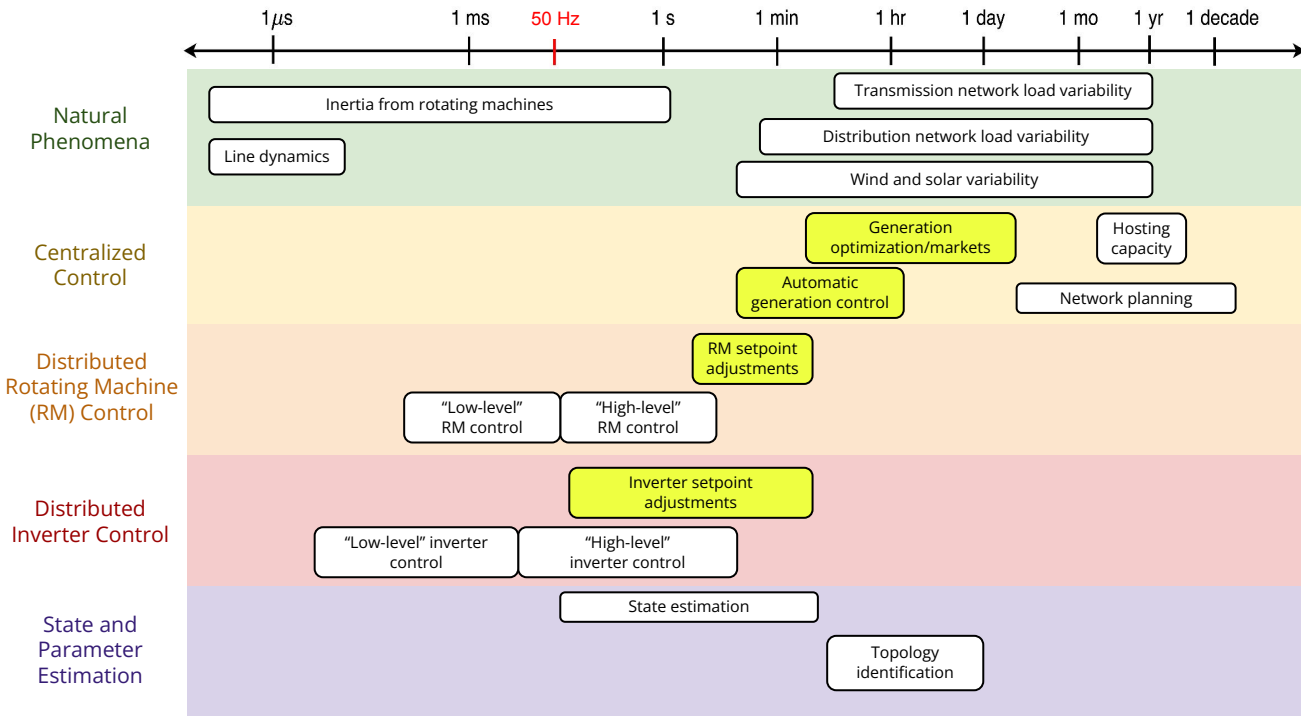
# Quasi steady state control



$$\min_{P, Q} f(P, Q)$$

s.t. Power flow equations,  
Network flow (thermal) constraints,  
Network bus (voltage) constraints,  
Individual generator constraints

# Primary, Secondary, and Tertiary control



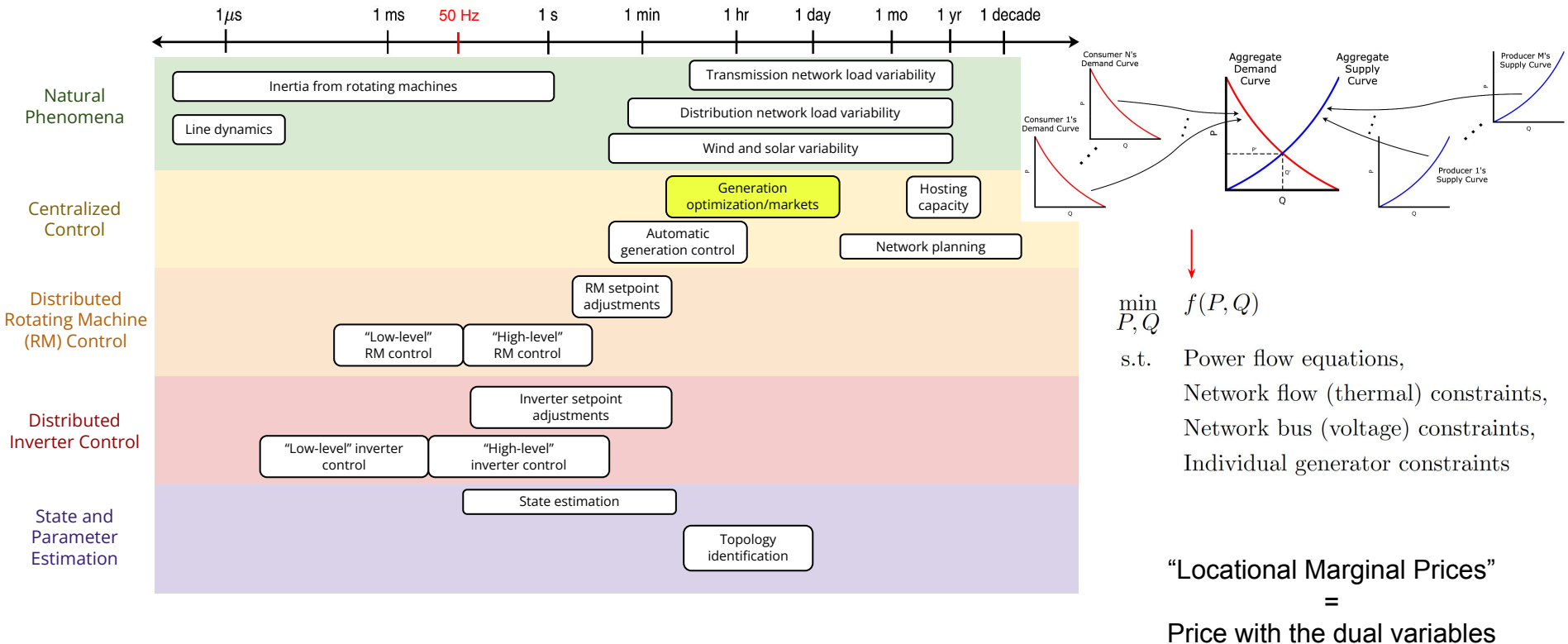
$$\min_{P, Q}$$

$$f(P, Q)$$

s.t.

- Power flow equations,
- Network flow (thermal) constraints,
- Network bus (voltage) constraints,
- Individual generator constraints

# Power systems markets



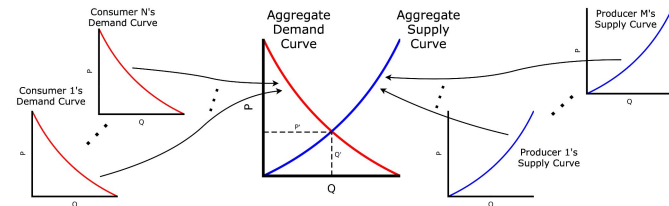
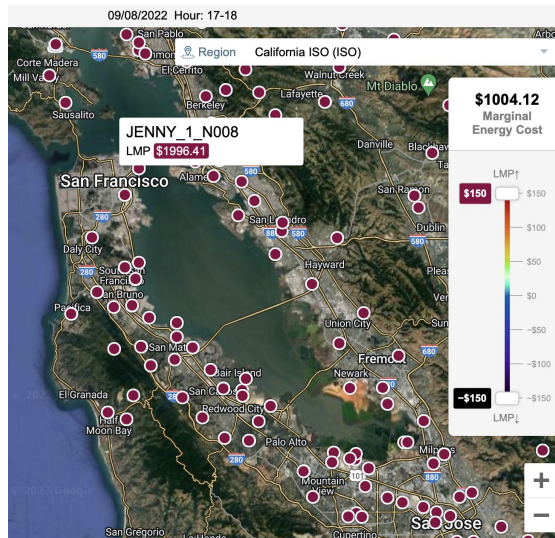
# Power systems markets



HEAT WAVE

## California heat wave: Bay Area records its highest-ever temperature

The worst of California's heat wave has arrived for the final day of the Labor Day weekend. Follow the latest news on the weather across the S. F. Bay Area and how it's impacting traffic, parks, the power grid and more.

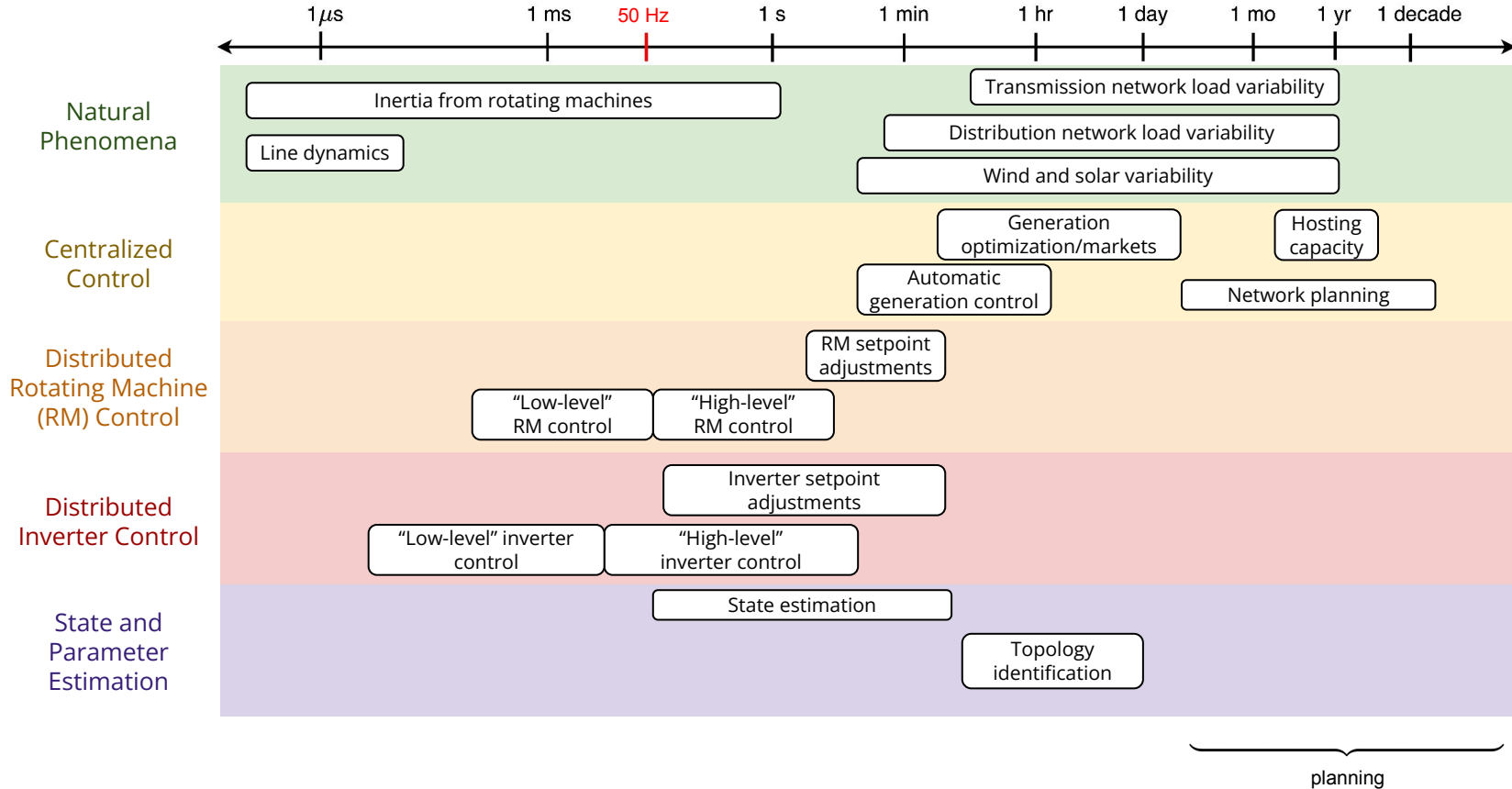


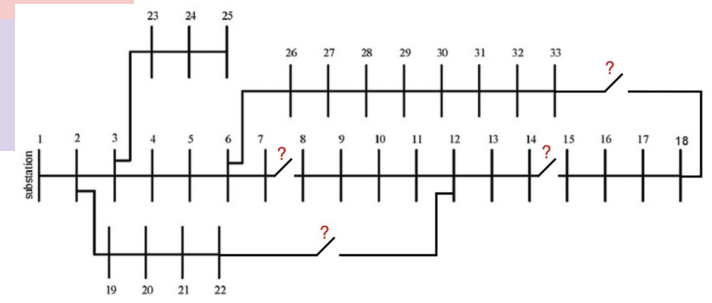
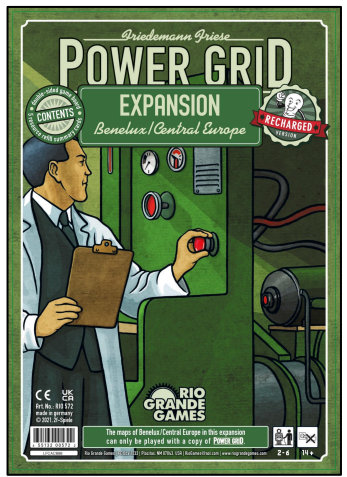
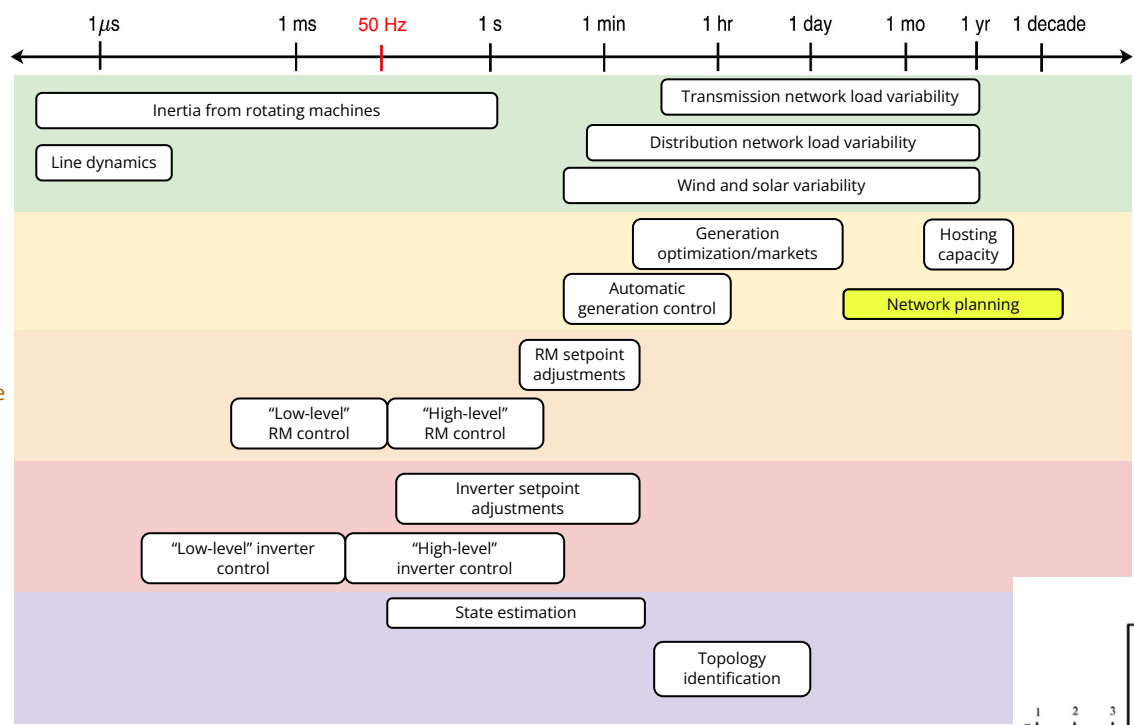
$$\min_{P, Q} f(P, Q)$$

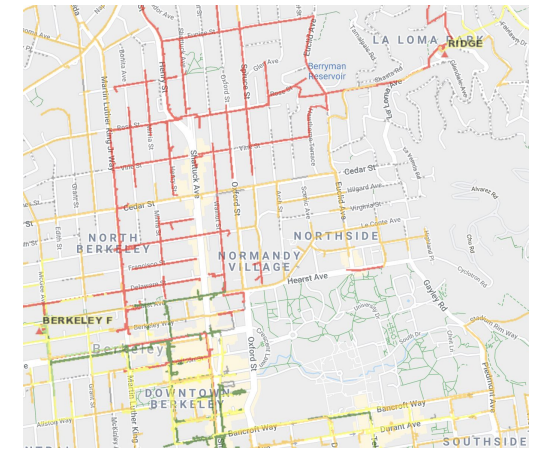
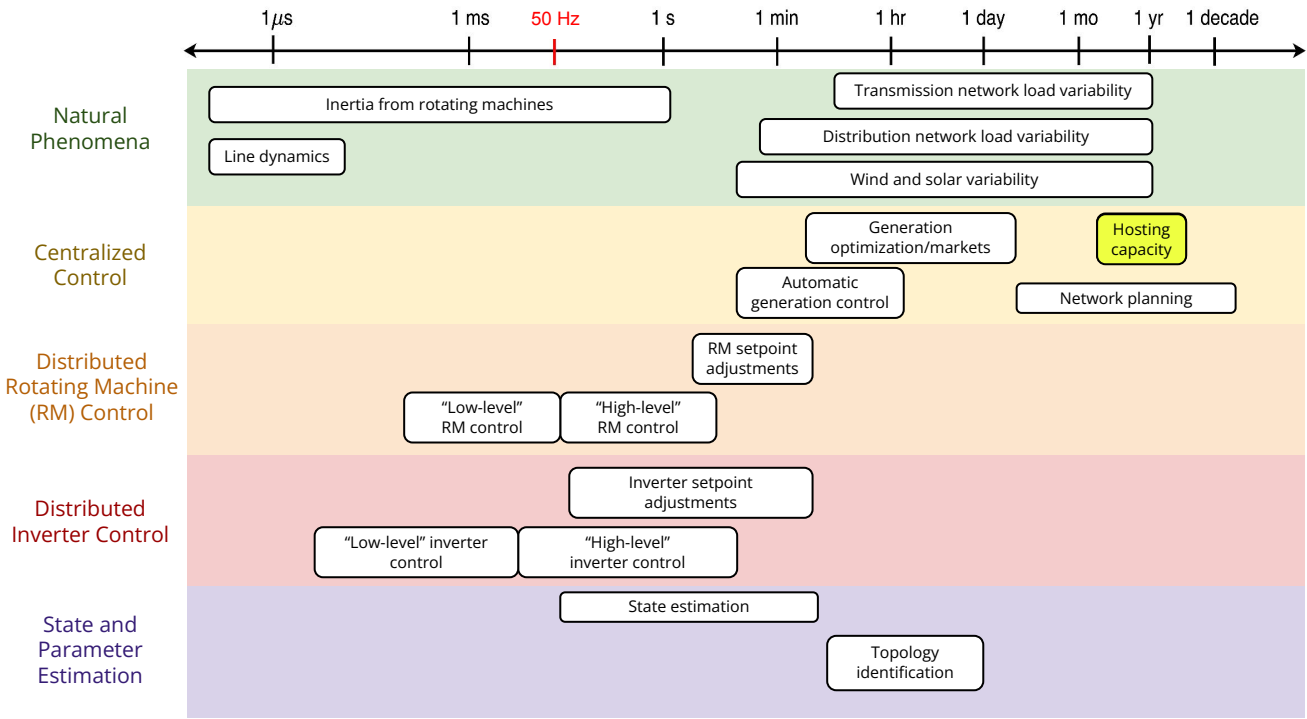
- s.t.
- Power flow equations,
  - Network flow (thermal) constraints,
  - Network bus (voltage) constraints,
  - Individual generator constraints

“Locational Marginal Prices”  
=  
Price with the dual variables

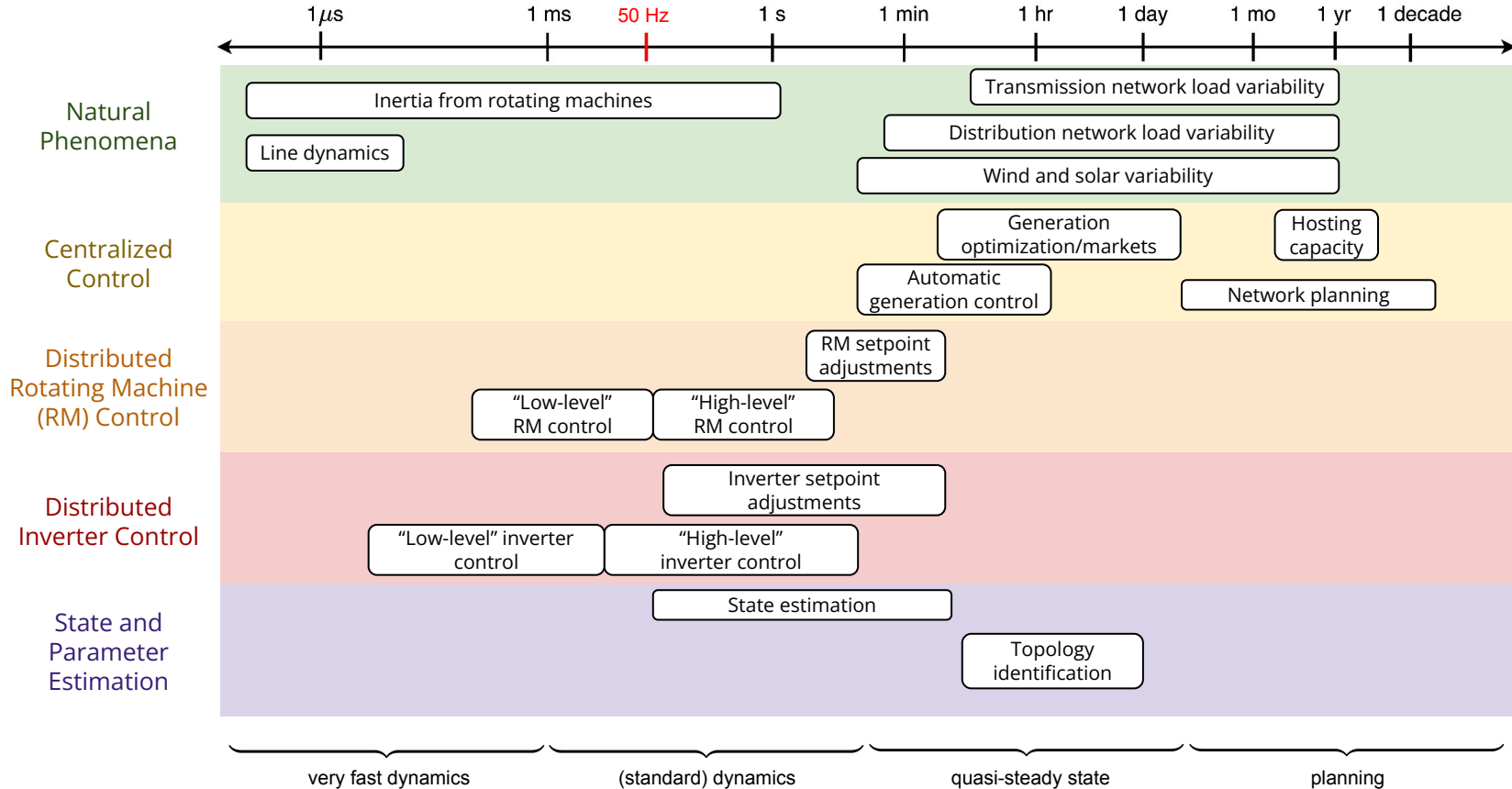
# Power Systems Timescales







# Power Systems Timescales





## Sources and image credits

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- [Others that I've forgotten, please send me an email to include a reference if you see one that is missed – kmoffat [at] ethz [dot] ch]