Introduction to Electric Power Systems

Review Lecture

Keith Moffat

1 Energy and Power

Energy: The capacity of something to do work, usually measured in Wh or J ($1J = 1Ws$).

Power: The rate at which energy is transferred from one object to another, usually measured in W. Mathematically, it is the time derivative of energy:

$$P = \frac{dE}{dt}$$

In rotational motion, power is measured as torque times rotational velocity: $T\omega$.
In electric circuits, power is measured as voltage times current: VI^*.

Mechanical power is converted to electrical power by an Electromotive Force (EMF), usually created by a generator. In the electrical domain, EMF manifests as a voltage.

Q. *What are the two ways that electric power can leave an electric circuit?*

Q. *In the field of power systems, why do we focus on power and not current? (even though the equation for power is nonlinear, and analyzing nonlinear systems of equations is more challenging than analyzing linear systems of equations.)*

Q. *Why does the electric grid use AC power?*

2 Phasors

Voltage and current phasors are the complex representation of a snapshot of voltage and current waveforms at a specific frequency f. For power systems, the frequency is almost always the 50 or 60 Hz nominal frequency.

Take the single frequency signal:

$$A\cos(\omega t + \theta) = \text{Real}\big(Ae^{j(\omega t+\theta)}\big)$$

$Ae^{j(\omega t+\theta)}$ can be broken into $Ae^{j\theta}e^{j\omega t}$, and if we remember that we are dealing with AC signals at the frequency ω, we can discard the $e^{j\omega t}$ term. That leaves us with the phasor:

$$Ae^{j\theta}$$

From our knowledge of Euler's formula, we recognize A as the amplitude and θ as the angle of the phasor in the complex plane. The time domain signal can always be retrieved from the phasor by multiplying the phasor by $e^{j\omega t}$ and taking the real portion of the signal.

The complex phasor $Ae^{j\theta}$ can also be converted to rectangular coordinates:

$$Ae^{j\theta} = A\cos\theta + jA\sin\theta.$$

The rectangular formulation is useful when adding phasors, while the complex exponential is useful when multiplying phasors.

Q. *By using complex phasors to represent the AC voltage and current waveforms, what do we not have to solve?*

3 Real, Reactive, and Apparent Power

Any current wave can be decomposed into a current wave that is in phase with the corresponding voltage wave, and a current wave that is orthogonal (ninety degrees out of phase) with the voltage wave.

Real power is the power that oscillates back and forth due to the portion of the current wave that is in phase with the voltage wave. Because the waves are in phase, and the real power wave touches zero each cycle, the average value of the power wave will also be equal to P.

Reactive power is the power that oscillates back and forth due to the current wave that is orthogonal (90 degrees shifted) to the voltage wave. Because this current wave is ninety degrees out of phase with the voltage, and power is the product of voltage and current, the reactive power wave will have an average value of 0.

Apparent power is the sum of the orthogonal real and reactive power waves, and describes the power of the aggregate current wave (which is equal to the sum of the in phase current wave and the orthogonal current wave). Because the reactive power wave has an average power of zero, the average of the apparent power is P.

3.1 Properties of Apparent Power

- $S = P + Qj = VI^*$
- S contains the information for the instantaneous power wave, which is a wave with twice the frequency of the voltage and current waves and a vertical offset. It is a small abuse of notation to put the complex S vector on the same complex plane as V, I, and Z (which is defined for phasors with frequency ω or constants), however putting S on the same complex plane is still generally done because it is convenient.
- $|S| = \sqrt{P^2 + Q^2}$ is the peak amplitude of the instantaneous power wave, not an RMS quantity.

3.2 Properties of Reactive Power

- Reactive power is measured in **Volt-Ampere Reactive (VARs)**. A VAR is the same size as a Watt, but the different terminology allows us to keep track of them separately. This is a semantic distinction that represents the physical separation between real and reactive power.
- Reactive power, like real power, is conserved across the system.

Q. *Why do we care where reactive power is generated?*

4 Generators, the Swing Equation, Transient Stability, and the Equal Area Criterion

To assess the transient stability of generators, we need both a mechanical and an electrical model for the generator. The most simple model for a generator (which we use by default) is the Thevenin equivalent circuit:

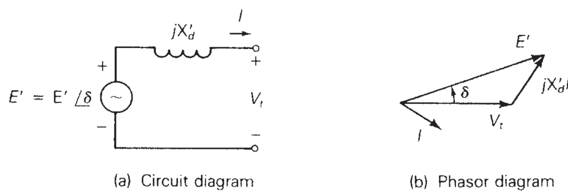


FIGURE 11.2 Simplified synchronous machine model for transient stability studies

For constant voltage magnitude, the real power transmitted across a purely inductive line is a function of δ:

$$P_e(\delta) = \frac{EV}{X}\sin(\delta) \tag{1}$$

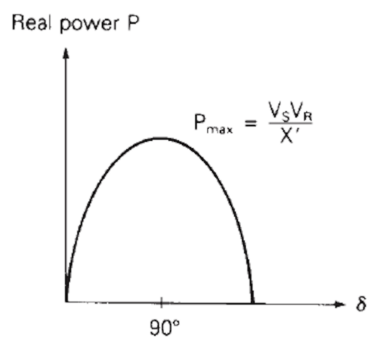


Figure 1: Real power delivered by a lossless line versus voltage angle across the line

4.1 The Swing Equation

The swing equation relates the mechanical power and rotations (oscillations in space) to the electrical power and oscillations in time. It describes how the net power into the machine's rotor determines the angle of the rotor.

Note: *For this class, δ is both the mechanical/rotational angle and the electrical angle of the EMF voltage source E. Understanding this equivalence between the angle of the rotor in space and the angle of the voltage wave in time is critical to understanding the swing equation and the equal area criterion.*

$$P_{\text{net}} = J\omega\ddot{\delta} \tag{2}$$

We introduce an additional approximation that $\omega \approx \omega_s$. While it is not necessary to set $\omega = \omega_s$, it makes the analysis easier. In addition, we group the constants together, and convert the equation to per unit (power base = S_B). Defining $H = \frac{\frac{1}{2}J\omega_s^2}{S_B}$ (the "normalized inertia constant"):

$$P_{\text{net}} = \frac{2H}{\omega_s}\ddot{\delta} \tag{3}$$

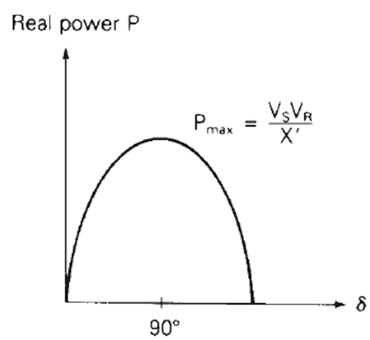
4.2 (Simplified) Rotor Transient Stability Analysis

Rotor transient stability analysis describes the position of the rotor after a (transient) disturbance affects the system, based on the net power exerted on the rotor. In order to simplify the transient stability analysis, power system engineers often make the following assumptions:

- $P_D = 0$, and therefore $P_{\text{net}} = P_m - P_e$. If the oscillations around δ^* are stable when we ignore P_D, then we know that the system will settle back to δ^* if P_D *is* included because P_D is a purely dissapative force.
- The transient disturbance is either a sudden change in P_m or a sudden change in the $P_e(\delta)$ curve. The sudden change will change the equilibrium angle δ^* and result in $P_{\text{net}} \neq 0$.
- P_m is constant throughout the duration of the transient, while P_e adjusts instantaneously according to Eqn. (1).

Q. *Why is it okay to assume that mechanical power extracted from the rotor is constant, while electrical power extracted from the rotor is not constant?*

To understand rotor transient stability analysis, we return to Fig. 1, pasted below for convenience:



Q. *Right after the sudden increase in P_m, is δ greater than, less than, or equal to δ^*?*

Q. *Right after the sudden increase in P_m, which direction does P_{net} initially push δ? (To the right or to the left in Fig. 1)*

Q. *What will happen when δ reaches δ^*? Will it stop or keep going?*

Q. *When δ has passed δ^*, which direction does P_{net} push δ? (To the right or to the left)*

The restorative force described by the questions above is the reason that synchronous machines are robust to small transient disturbances, and are the key idea behind the equal area criterion.

4.3 The Equal Area Criterion

The equal area criterion is a transient stability criterion for a single machine, modeled using the swing equation, attached to an infinite bus (or another generator). The equal area criterion determines if the system will return to the same operating point after a disturbance, or slip a pole and diverge to a different operating point.
The equal area criterion:

The area that corresponds to the acceleration of the rotor on the δ–P_e graph is equal to the area that corresponds to the deceleration.

This phenomenon is described well by the figure from page 830 of the classic Power System Stability and Control text by Kundur. In order to understand this figure, we need to define some new variables:

- P_{m0} is the mechanical power applied before the step change in mechanical power.
- P_{m1} is the mechanical power applied after the step change in mechanical power.
- δ_0 is the stable operating angle before the step change in mechanical power.
- δ_1 is the stable operating angle after the step change in mechanical power (δ^* above).
- δ_m is the maximum angle that the rotor will swing to, according to the equal area criterion.
- δ_L is the angle at which $P_e = P_{m1}$ on the right side of the δ–P_e curve.

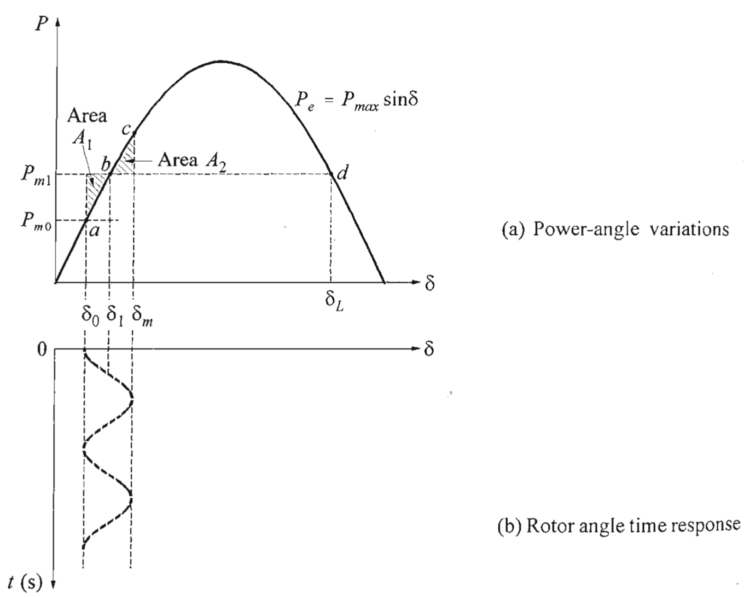


Figure 13.4 Response to a step change in mechanical power input

Q. *Why is δ_L important?*

Q. *Intuitively, why does the equal area criterion hold?*

5 Bus Admittance Matrix

The complex-valued nodal admittance matrix Y relates the vector of complex voltages at each bus to the vector of current *injections* at each bus:

$$I = YV \tag{4}$$

Y contains *both* the network connectivity and the impedance information of the network, in one intuitively understood matrix.

If $Y(i,k)$ is the $(i,k)^{\text{th}}$ entry of Y, the admittance of a line between nodes i and k is y_{ik}, and the shunt admittance at node i is y_i, Y is constructed using the following rules:

$$Y(i,k) = \begin{cases} y_i + \sum_{l \neq i} y_{il}, & \text{if} \quad i = k \\ -y_{ik}, & \text{if} \quad i \neq k \end{cases} \tag{5}$$

An important subtlety is that the current injections in the I vector are in *parallel* with the current flowing through the shunt admittance y_i.

Q. *What is value of the $(i,k)^{\text{th}}$ entry of Y when the i and k nodes aren't connected? Is this what you would expect?*

Q. *What is the value of y_i when no shunt admittance is attached to bus i? Is this what you would expect?*

6 Power Flow

6.1 Power Flow Equations

Defining $\text{diag}(V)$ as a matrix with vector V in the diagonal entries and zeros elsewhere, the nodal power injections S are given by:

$$\begin{aligned} S &= \text{diag}(V) I^* \\ &= \text{diag}(V)(YV)^* \end{aligned} \tag{6}$$

These equations are nonlinear because of the VV^* multiplication.
Equivalently, these equations can be written using only real-valued variables:

$$
\begin{aligned}
P_i = P_{Gi} - P_{Di} &= \sum_{k=1}^{N} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) \\
Q_i = Q_{Gi} - Q_{Di} &= \sum_{k=1}^{N} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})
\end{aligned} \tag{7}
$$

Both (6) and (7) encode the same information about the relationship between the power injections and the voltages on the network, they just do so in different coordinates.

Q. *Why would we want rectangular coordinates for power, and polar coordinates for voltage?*

6.2 Power Flow

Power flow determines the voltages on the network, given the following information for each bus:

- Loads: P and Q
- Generators: P and Q, or P and $|V|$
- Slack bus: $|V| = 1$ and $\delta = 0$

The slack bus serves two purposes in power flow:

1. It injects the necessary real and reactive power so that the power balance is met for the network, including network losses.
2. It serves as the phasor angle reference.

Determining which bus serves as the slack bus in power flow analysis is often a design decision because rarely does a single node satisfy the network power imbalance in reality.[1]

Q. *Why do we need a slack bus for power flow to converge?*

Power flow is a system of nonlinear equations, and therefore hard to solve.

6.3 Newton's Method For Power Flow

$$f(x) = g(x) - [P; Q]. \tag{8}$$

We have found a solution to $f(x)$ when (8) is equal to zero.

Recall the power flow problem for just two buses. This problem can have zero, one or two voltage solutions. With three buses there can be between zero and four solutions. In general the number of solutions is upper bounded exponentially by the number of nodes N: (number of solutions) $< 2^{N-1}$. Finding the

[1] One exception to this paradigm is the substation node in distribution networks, which behaves like an infinite bus for the distribution network if the transmission line impedances are negligible compared with the distribution line impedances.

high voltage solution(s) can be challenging, and is dependent on the initial guess. If a solution is found, it is possible that the solution is a low voltage solution (recall the two bus example), and not actually useful for the network. If this occurs, another initial guess must be tried.

It is also possible that there are no solutions, as we saw for general nonlinear equations in Section **??**. When there are no solutions, Newton's Method will not converge no matter what initial guess is used.

Q. *What does it mean when the Newton method for power flow doesn't converge?*

Hint: In what scenarios might power flow not converge?
Without additional information, is it possible to distinguish between these scenarios?

When learning power flow, it is important to run Newton's method (and/or Gauss-Seidel) by hand at least once to get intuition for how the algorithm works. In practice, though, a power system engineer would never execute iterative power flow by hand, and probably not have to code up an iterative algorithm either because commercial power flow solvers exist. It is, however, important to understand what is going on under the hood so you can know what to expect. Specifically, it is important to understand:

- Why iterative solution methods are used
- Why a slack bus is needed
- That power flow can have multiple solutions (i.e. high and low voltage solutions), or no solution
- That the initial guess affects power flow convergence
  - The initial guess will determine which power flow solution the iterative method will converge to
  - For some initial guesses iterative power flow will not converge, even though the power injections are feasible. It is not easy to differentiate this circumstance from the circumstance in which the power injections are not feasible