Introduction to Electric Power Systems
Lecture 9
Rotor/Angle Transient Stability

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1 Thevenin Equivalent and Max Power Transfer Review

To develop the swing equation and the equal area criterion we need both a mechanical and an electrical model for the generator. We will cover the mechanical model in the next section. For the electrical model, we will use the Thevenin Equivalent generator model which has an internal EMF voltage source E, and an inductive output impedance X that models the inductance of the motor coils:

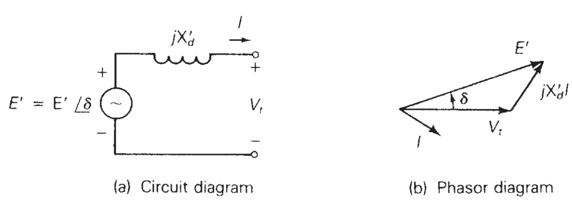


FIGURE 11.2
Simplified synchronous machine model for transient stability studies

For the equal area criterion analysis, we attach the Thevenin equivalent generator model to an infinite bus (constant AC voltage source).The infinite bus voltage source V_s serves as the voltage reference and thus $\angle V_s = 0$. The frequency of the infinite bus voltage ω_s is constant. δ is the angle of the EMF voltage source of the machine, relative to the infinite bus.

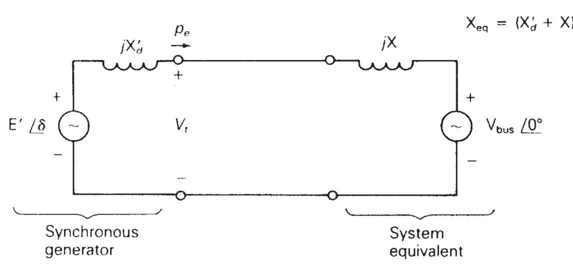


FIGURE 11.3
Synchronous generator connected to a system equivalent

For constant voltage magnitude, the real power tranmsitted across a purely inductive line is a function of δ:

$$P_e(\delta) = \frac{EV}{X}\sin(\delta) \tag{1}$$

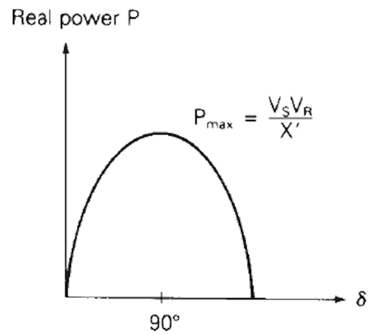


Figure 1: Real power delivered by a lossless line versus voltage angle across the line

When the system is operating at steady state (what we've analyzed up to this point in the class), the power injected into the rotor by the prime mover and the power extracted from the rotor in the form of electric power (and machine losses) are equal. The machine rotates at a constant rotational velocity that matches the grid frequency, and angle of the machine is constant in time.

When a disturbance occurs, such as a change in the load, a change in the power from the generator's power prime mover, or a change in the network, the system is *not* at steady state. The power injected into the rotor by the prime mover and the power extracted from the rotor are *not* equal, and the machines rotational velocity and angle will be transient (*not* constant in time).

2 The Swing Equation

The swing equation relates the mechanical power and rotations (oscillations in space) to the electrical power and oscillations in time. It describes how the net power into the machine's rotor determines the angle of the rotor. The equations of motion are differential equations (e.g. F = Ma), thus the swing equation will be a differential equation.

The swing equation is derived from the following variables, (though these variables are not what are used in the canonical form):

- τ_m: The mechanical torque (rotational force) applied to the rotor by the prime mover
- τ_D: The dampening torque
- τ_e: The mechanical torque applied to the rotor due to the electric power that is extracted from the machine
- $\tau_{\text{net}} = \tau_m - \tau_e - \tau_D$: the net torque (force) applied to the rotor
- J: the rotational moment of inertia of the rotor
- θ: the absolute angle of the rotor (in a non-rotating reference frame)
- α: the angular acceleration of the rotor, $\alpha = \ddot{\theta}$

Newton's equation for rotational motion states that:

$$\tau_{\text{net}} = J\alpha \tag{2}$$

This equation is the basis for the swing equation. In addition, we define the following variables:

- δ: the phase shift of the rotor (angle of the rotor in the rotating reference frame)
- ω: the rotational velocity of the rotor, $\omega = \dot{\delta}$
- ω_s: the angle reference rotational velocity

The **first** modification is describing α in terms of δ rather than θ We prefer δ because δ is also the electrical angle, which determines the real power that is extracted from the machine according to Eqn. (1). The equation relating the angle θ in the stationary reference frame to the angle δ in the rotating reference frame is:

$$\theta = \omega_s t + \delta.$$

Unlike δ, θ, ω, and α, ω_s is a constant and *not* a function of time. By taking the derivative and second derivative of both sides we get:

$$\dot{\theta} = \omega_s + \dot{\delta} \tag{3}$$

$$\ddot{\theta} = \ddot{\delta} \tag{4}$$

We see that $\ddot{\theta} = \ddot{\delta} = \alpha$. Thus, no modifications are necessary to define Eqn. (2) in terms of δ.

Note: *δ is both the mechanical/rotational angle and the electrical angle of the EMF voltage source E. Understanding this equivalence between the angle of the rotor in space and the angle of the voltage wave in time is critical to understanding the swing equation and the equal area criterion.*

The **second** modification describes the swing equation in terms of the power injections, rather than the torques.

- P_m: The power from the mechanical torque (rotational force) applied to the rotor by the prime mover
- P_D: The power from the dampening torque
- P_e: The power from the mechanical torque applied to the rotor due to the electric power that is extracted from the machine
- $P_{\text{net}} = P_m - P_e - P_D$: the net power transferred to the rotor

Multiplying both sides of Eqn. (2) by ω and using the equation for rotational power $P = \tau\omega$ gives:

$$P_{\text{net}} = J\omega\ddot{\delta} \tag{5}$$

From this equation it is evident that the swing equation models how the angle of the machine changes with time due to the net power that is delivered to the rotor P_{net}—if $P_{\text{net}} > 0$ the rotor will accelerate. If $P_{\text{net}} < 0$ the rotor will decelerate.

The **third** modification makes the approximation that $\omega \approx \omega_s$. While it is not necessary to set $\omega = \omega_s$, it makes the analysis easier. In addition, we group the constants together, and convert the equation to per unit (power base = S_B). Defining $H = \frac{\frac{1}{2}J\omega_s^2}{S_B}$ (the "normalized inertia constant"):

$$P_{\text{net}} = \frac{2H}{\omega_s}\ddot{\delta} \tag{6}$$

The swing equation is a differential equation. The equilibrium point δ^* of Eqn. (6) is given by the point at which the net power delivered to the rotor P_{net} is zero. Ignoring P_D, δ^* is determined by the intersection of P_m and the function for electrical power $P_e(\delta)$—as P_m changes, the equilibrium point will change.

Q. *Is the system at rest when $\delta = \delta^*$, $P_{\text{net}} = 0$, and $\dot{\delta}(t) = 0$?*

Q. *Is the system at rest when $\delta = \delta^*$, $P_{\text{net}} = 0$, and $\dot{\delta}(t) \neq 0$?*

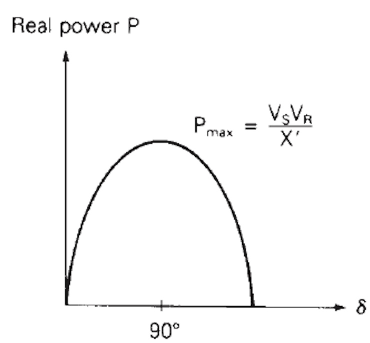
3 (Simplified) Rotor Transient Stability Analysis

Rotor transient stability analysis describes the position of the rotor after a (transient) disturbance affects the system, based on the net power exerted on the rotor. In order to simplify the transient stability analysis, power system engineers often make the following assumptions:

- $P_D = 0$, and therefore $P_{\text{net}} = P_m - P_e$. If the oscillations around δ^* are stable when we ignore P_D, then we know that the system will settle back to δ^* if P_D *is* included because P_D is a purely dissapative force.
- The transient disturbance is either a sudden change in P_m or a sudden change in the $P_e(\delta)$ curve. The sudden change will change the equilibrium angle δ^* and result in $P_{\text{net}} \neq 0$.
- P_m is constant throughout the duration of the transient, while P_e adjusts instantaneously according to Eqn. (1).

Q. *Why is it okay to assume that mechanical power extracted from the rotor is constant, while electrical power extracted from the rotor is not constant?*

To understand rotor transient stability analysis, we return to Fig. 1, pasted below for convenience:



Q. *Right after the sudden increase in P_m, is δ greater than, less than, or equal to δ^*?*

Q. *Right after the sudden increase in P_m, which direction does P_{net} initially push δ? (To the right or to the left in Fig. 1)*

Q. *What will happen when δ reaches δ^*? Will it stop or keep going?*

Q. *When δ has passed δ^*, which direction does P_{net} push δ? (To the right or to the left)*

The restorative force described by the questions above is the reason that synchronous machines are robust to small transient disturbances, and are the key idea behind the equal area criterion.

4 The Equal Area Criterion

The equal area criterion is a transient stability criterion for a single machine, modeled using the swing equation, attached to an infinite bus (or another generator). The equal area criterion determines if the system will return to the same operating point after a disturbance, or slip a pole and diverge to a different operating point.

The equal area criterion:

The area that corresponds to the acceleration of the rotor on the δ–P_e graph is equal to the area that corresponds to the deceleration.

This phenomenon is described well by the figure from page 830 of the classic Power System Stability and Control text by Kundur. In order to understand this figure, we need to define some new variables:

- P_{m0} is the mechanical power applied before the step change in mechanical power.
- P_{m1} is the mechanical power applied after the step change in mechanical power.
- δ_0 is the stable operating angle before the step change in mechanical power.
- δ_1 is the stable operating angle after the step change in mechanical power.
- δ_m is the maximum angle that the rotor will swing to, according to the equal area criterion.
- δ_L is the angle at which $P_e = P_{m1}$ on the right side of the δ–P_e curve.

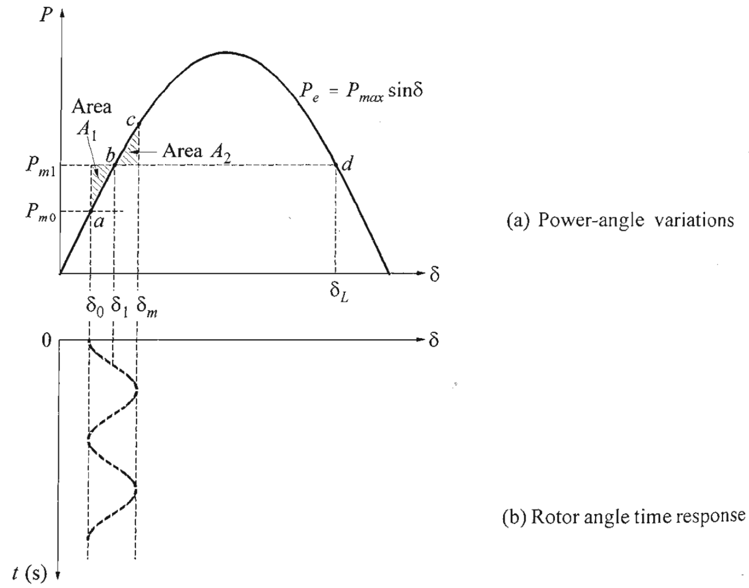


Figure 13.4 Response to a step change in mechanical power input

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Q. *Why is δ_L important?*

Q. *Intuitively, why does the equal area criterion hold?*

One of the things that is initially surprising about the equal area criterion is that the x axis for the area is the angle δ, rather than time t. You can integrate the net power injection over the angle just as well as you can integrate the net power injection over time, though, as demonstrated by the equal area criterion derivation in Appendix A. The equal area criterion uses the angle δ for the x axis because equation (1) expresses the electrical power P_e in terms of δ, and because using the angle δ for the x axis provides useful intuition.

Note that the figure from Kundur does not include the affects of dampening. If the oscillations are stable without dampening, then including dampening in the system will reduce the magnitude of each oscillation until the system is at rest at the steady state equilibrium. The dampened oscillations are classic stable dynamic system behavior.

However, if the initial perturbation is too large, the rotor angle may go past the point where P_e and P_m intersect and the restorative force switches to a divergent force. The rotor, which had previously been slowing down, will start to speed up again, away from δ^*. In the best case scenario, the rotor will settle into a new stable equilibrium. This is called slipping a pole. Even if a new stable equilibrium is found, the rotor will move violently in a way that it was not designed to. In the worst case scenario the rotor will keep accelerating until it breaks.

In summary, the combination of the swing equation and the equation for $P_e(\delta)$ creates a restorative force *as long as* $\delta < \delta_L$. If $\delta > \delta_L$ then the restorative force becomes a divergent force and the generator slips a pole and bad things happen. If, for a given disturbance, there is not enough deceleration area available to the left of δ_L, then the system is in an unstable operating state. So you know you are going to slip a pole before you actually slip a pole. This phenomenon can be described well by the Todd Snider lyric from the song 45 Miles:

"Now some of you who that have been in a car accident can testify to the fact that there is a brief moment between realizing you're going to crash your car... and crashing your car."

There are three types of disturbances commonly encountered in equal area criterion analysis:

1. a sudden change in P_m,
2. a line fault for which the P_e temporarily goes to zero, before the fault is cleared, and
3. a reconfiguration of the network, which changes the $P_e(\delta)$ curve.

The Kundur image above is an example of the first kind of disturbance, in which P_m changes. In disturbances (2) and (3), it is usually assumed that P_m does not change.

Q. *What does the δ–P_e curve look like for disturbances of the types (2) and (3)?*

A Equal Area Criterion Derivation

Note: you do not have to understand this derivation of the equal area criterion for this class. I am just putting it here for students who might be interested.

To derive the equal area criterion, we integrate the swing equation (6) over the angle δ from the initial angle δ_0 to the maximum swing angle δ_m:

$$\int_{\delta_0}^{\delta_m} \frac{2H}{\omega_s} \frac{d^2\delta}{dt^2} d\delta = \int_{\delta_0}^{\delta_m} (P_m - P_e) d\delta \tag{7}$$

In order to evaluate the integral on the left hand side, we set $u = \left[\frac{d\delta}{dt}\right]^2$, and (taking advantage of Liebniz derivative notation) observe that

$$\begin{aligned} \frac{du}{d\delta} &= 2\left[\frac{d\delta}{dt}\right] \frac{d}{d\delta}\left(\frac{d\delta}{dt}\right) \\ &= 2\left[\frac{d\delta}{dt}\right]\left(\frac{d^2\delta}{d\delta dt}\right) \\ &= 2\left(\frac{d^2\delta}{dt^2}\right), \\ \text{and } du &= 2\left(\frac{d^2\delta}{dt^2}\right) d\delta. \end{aligned}$$

Substituting du into (7), we get

$$\frac{H}{\omega_s} \int_{\delta_0}^{\delta_m} du = \int_{\delta_0}^{\delta_m} (P_m - P_e) d\delta \tag{8}$$

$$\frac{H}{\omega_s} u \Big|_{\delta_0}^{\delta_m} = \int_{\delta_0}^{\delta_m} (P_m - P_e) d\delta. \tag{9}$$

$u = \left[\frac{d\delta}{dt}\right]^2$ is the squared rate of change of δ. We know that δ is not moving at the moment it starts, δ_0, and at the moment at which it switches directions, δ_m. Therefore, the integral on the left hand side of (9) will be equal to zero. This leaves

$$\int_{\delta_0}^{\delta_m} (P_m - P_e) d\delta = 0.$$

Splitting the integral at δ_1, we get the equal area criterion:

$$\int_{\delta_0}^{\delta_1} (P_m - P_e) d\delta = \int_{\delta_1}^{\delta_m} (P_m - P_e) d\delta. \tag{10}$$