Introduction to Electric Power Systems
Lecture 7
Transformers and Per Unit

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1 Transformers

Transformers are ubiquitous in AC power grids. They are stationary devices that convert AC power from one voltage domain to another voltage domain.[1] In fact, transformers are the reason the grid is AC, rather than DC. Transformers allow AC power, generated at a voltage optimized for the generator, to be converted to a higher voltage domain for transmission (to mitigate transmission losses), then converted back down to a lower voltage domain for end-use.

A transformer (usually) has two sides, primary and secondary. It transmits energy from one side to the other via a magnetic flux in a shared transformer core. Thus, the two sides of the transformer are not electrically connected at all, and two "voltage domains" are created. The two voltage domains can be either single phase or multi-phase.

Faraday's law (and Lenz's Law) describe the relationship between and induced voltage and a *change* in (linked) magnetic flux:

$$E = -N\frac{d\phi}{dt} \tag{1}$$

The electromotive force (voltage) E induced by Faraday's law is also accompanied by a reactionary "magnetomotive force" (MMF) force that *opposes* the change in the magnetic field.

Take the primary-to-secondary relationship (though the secondary-to-primary relationship works the same)—applying a voltage across the primary transformer winding will induce a current in the primary winding. This current will create a MMF on the core, which will try to induce magnetic flux lines in the core. The relationship between the MMF created by the primary winding (N_1I_1), the secondary winding (N_2I_2) and the magnetic flux in the core Φ can be described by the following magnetic "circuit," derived from Ampere's Law:

$$N_1I_1 - N_2I_2 = \mathcal{R}_c\Phi_c$$

$\mathcal{R}_c = \frac{l_c}{\mu_c A_c}$ is the "reluctance" of the magnetic core. If the core has *infinite* permeability μ_c, $\mathcal{R}_c$ will be zero and the initial MMF and reactionary MMF will be exactly equal. Thus, no magnetic flux will ever be created in the core ($\Phi = 0$) and the two windings will be coupled exactly, according to the turns ratio a.

[1] They are useful for other reasons as well, such as galvanic isolation, but this is their main use in power systems.

The voltage is inversely proportional to the turns ratio $a = \frac{N_1}{N_2}$, while the current is proportional to the turns ratio.

$$V_2 = \frac{1}{a} V_1, \tag{2}$$

$$I_2 = a I_1 \tag{3}$$

These voltage and current relationships can be derived from Faraday's Law and Ampere's Law, respectively.

Q. *Demonstrate that the equations (2) and (3) adhere to conservation of energy:*

Q. *If a DC voltage (or current) source was placed across the primary side of a transformer a long time ago and has not been changed at all, what do the secondary side's windings notice at the present moment? Why?*

Q. *If a transformer core has infinite permeability (impossible, in reality), what is the magnetic flux in the transformer core when the primary side current crosses zero? And what is the flux when the primary side current is at it's max value?*

Q. *If the secondary winding of a transformer is attached to an open, corresponding to infinite impedance, the transformer just looks like an inductor to the primary side. If the transformer core has infinite permeability, and the secondary winding is attached to an open, what inductance does the primary side see? Would it be possible to push current through the primary winding?*

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2 Transformer Polarity

The direction in which a wire is wrapped around the core determines if the flux goes up or down. What we are interested in with a transformer isn't whether the flux goes up or down, but rather what the relationship between the direction of the two windings is. The dot method ("polarity marks") makes all of this much easier. Without the dot convention, you would need to look at the direction of the windings (clockwise or counterclockwise), use the right hand rule and remember Lenz's Law.

With the dot convention, you only need to learn two rules of thumb. First, some explanation. One dot is placed per winding (one on each side, in the case of a single phase transformer) such that when current enters the transformer at an entry with a dot, the mmf in the core aligns with the mmf created by current entering at the dot on the other side of the transformer.

If both voltages are defined such that both positive (or negative) voltage polarities are collocated with dots, then evaluating Faraday's law for each winding (and assuming all the magnetic flux is shared) gives the following equation:

$$\frac{V_1}{N_1} = \frac{d\phi}{dt} = \frac{V_2}{N_2}$$

If the voltages is defined such that just one of the voltages has a positive polarity collocated with a dot (say the primary + is collocated with the dot, while the secondary - is collocated with the dot), then (assuming all the magnetic flux is shared,) we have the following equation:

$$\frac{V_1}{N_1} = \frac{d\phi}{dt} = -\frac{V_2}{N_2}$$

Polarity rule of thumb 1: *If both the positive or negative primary and secondary voltage polarities are collocated with dots, the following equation holds:* $\frac{V_1}{V_2} = a$*. If the polarities are switched, relative to the dots, the following equation holds:* $\frac{V_1}{V_2} = -a$*.*

Regarding current, Lenz's Law states that the direction of the current induced in a circuit due to a change in a magnetic field opposes the change in flux. Thus when current flows into the dot at one side, it will induce a current flowing *out* of the dot on the other side, because the reactionary current *opposes* the creation of the magnetic flux from the primary side.

Polarity rule of thumb 2: *If one current flows into a dot and one out of a dot, the following equation holds:* $I_2 = I_1 a$*. If both currents flow into or out of the dots, the following equation holds:* $I_2 = -I_1 a$*.*

The two rules of thumb indicate that, if you get to choose how voltages and currents are defined, it is best to define the voltages so that both positive (or negative) voltage polarities are collocated with dots, and that one current flows into a dot and the other current flows out of the other dot. If you define the voltages and currents this way, won't have to include negative signs when reflecting voltages and currents across the transformer.

3 Referred Impedance

Recall that transformers split a circuit up into different voltage domains. Impedance is defined as the ratio between ΔV and I for a given circuit component. The ratio between ΔV and I will be different though, when seen from a different voltage domain. For example, a resistor of impedance 1Ω will look different if it exists on the other side of a transformer that scales voltage and current by the turns ratio a. One way of addressing how transformers affect the impedances seen in a given voltage domain is to "refer" all of the impedances to the given voltage domain.

Q. *For an ideal transformer, what do we need to know to calculate the referred impedance?*

Q. *If I have impedance* Z *on the secondary side, derive an expression for the referred impedance* Z' *in terms of* Z *and* $a = \frac{N_1}{N_2}$*.*

Q. *A 10V voltage source supplies power to a 1Ω resistor, after the voltage has been stepped down by a transformer with turns ratio a. Is the power supplied by the source greater if $a = 10$, or $a = 11$? Describe why, using referred impedance.*

Q. *A power source supplies power to a 1A, constant current load. The transmission path goes through a transformer with turns ratio a, then a 0.1Ω line-impedance resistance, before reaching the load. Are the* losses *greater if $a = 10$, or $a = 11$? Describe why, using referred impedance.*

4 Per-Unit System

Power engineers use the per-unit system to simplify calculations on networks with transformers. (All large power networks have transformers.) The per unit system allows you to eliminate ideal transformers from your analysis by establishing "base" voltage, current, power and impedance values on the network. For a given quantity x:

$$x_{\text{per unit}} = \frac{x}{x_{\text{base}}}$$

x_{base} will always be *real-valued*, and thus per unit calculations only shift magnitude, not phase.

There are four primary per unit bases: $S_{\text{base},1\phi}$, V_{baseLN}, I_{base} and Z_{base}. LN means line-to-neutral, and 1ϕ means single phase. The equations relating the per unit bases for a single phase system are:

$$\begin{aligned}
P_{\text{base}1\phi} &= Q_{\text{base}1\phi} = S_{\text{base}1\phi} \\
I_{\text{base}} &= \frac{S_{\text{base}1\phi}}{V_{\text{baseLN}}} \\
Z_{\text{base}} &= R_{\text{base}} = X_{\text{base}} = \frac{V_{\text{baseLN}}}{I_{\text{base}}} = \frac{V^2_{\text{baseLN}}}{S_{\text{base}1\phi}} \\
Y_{\text{base}} &= G_{\text{base}} = B_{\text{base}} = \frac{1}{Z_{\text{base}}}
\end{aligned}$$

A power engineer is free to choose any two of the four base values. The two chosen values will determine the others. The common choices are V_{baseLN} and $S_{\text{base},1\phi}$. Per unit base values can be different on either side of a transformer. The convention is for $S_{\text{base},1\phi}$ to be shared throughout the network regardless of transformer ratios, while V_{baseLN} are altered by each transformer according to the transformer's turn ratio $a = \frac{N_1}{N_2}$:

$$\frac{V_{\text{base},1}}{V_{\text{base},2}} = a \tag{4}$$

Thus, V_{baseLN}, I_{base} and Z_{base} are different for each voltage domain. If this convention is used when calculating the per unit bases for the network, the ideal transformers on the network can be eliminated when conducting per unit analysis.

Q. *For a transformer with turns ratio a, derive an expression for $Z_{\text{base},2}$ in terms of $Z_{\text{base},1}$.*

Q. *If you have converted impedances to per unit, do you need to refer impedances across ideal transformers?*

Q. *A $10V$ voltage source supplies power to a 1Ω resistor, after the voltage has been stepped down by a transformer with turns ratio a. Is the power* supplied *greater if $a = 10$, or $a = 11$? Describe why, using per unit.*

Q. *A power source supplies power to a $1A$, constant current load. The transmission path goes through a transformer with turns ratio a, then a 0.1Ω resistor, before reaching the load. Are the* losses *greater if $a = 10$, or $a = 11$? Describe why, using per unit.*

The previous two questions are identical to the referred impedance questions, but solved with per unit impedances rather than referred impedances.

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