Introduction to Electric Power Systems
Lecture 6B

Transmission Line Physics

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1 Series Inductance in transmission Lines

Inductance L of a circuit element is defined as the ratio of the flux linkage λ over all space to the current I through the element, $L := \frac{\lambda}{I}$. Thus, inductance describes the ratio of the change in flux linkage to the change in the current, $L = \frac{d\lambda}{di}$. When we combine the definition of inductance with Faraday's law, $V = \frac{d\lambda}{dt}$, we get the circuit equation for inductors:

$$V = L\frac{di}{dt}. \tag{1}$$

For power systems, the equation for inductance L in terms of the distance between the two wires D, and the adjusted radius of the wire, $r' = e^{\frac{-1}{4}}r$, is:

$$L = \frac{d\lambda}{di} = 2 \times 10^{-7} \ln \frac{D}{r'}. \tag{2}$$

This equation is derived in A.

Q. *Annotate the description of the derivation of Eqn. (2) in Appendix A with three figures.*

Q. *Where did the* $2 * 10^-7$ *come from in equation 2?:*

Q. *Where the natural log (ln) come from?*:

Q. *Where did the D come from? Why is it in the numerator of the ln?*:

Q. *Where did the r' come from? Why is it in the denominator of the ln?*:

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2 More than Two Conductors

If we have more than two conductors (total), the solution approach is to represent the set of conductors as a single (fictitious) sending conductor and a single (fictitious) return conductor. This problem-reduction is done by finding the average flux linkage. The average flux linkage can be calculated by using the average of logarithms of distances. We use the geometric mean of distances (rather than the arithmetic mean of distances) because we are interested in the average of logarithms of distance. For example, $\frac{1}{3}(\ln(D_1) + \ln(D_2) + \ln(D_3)) = \frac{1}{3}\ln(D_1 D_2 D_3) = \ln((D_1 D_2 D_3)^{\frac{1}{3}}) = \ln D_{eq}$ if D_{eq} is the geometric mean of D_1, D_2, and D_3. The geometric mean is used both in situations in which there is more than one conductor for a given phase, and in situations in which there is more than one return line (i.e. three phase power).

The geometric mean equation is:

$$D_{\text{mean}} = \Big(\prod_{n=1}^{N} \prod_{m=1}^{M} D_{nm} \Big)^{\frac{1}{NM}} \tag{3}$$

Geometric Mean Radius (GMR): In situations in which the current of a given wire is carried by multiple conductors, we use D_{GMR} to give an "effective radius" for a single wire that would behave equivalently to the wire batch. For D_{GMR}, $N = M$ in equation 3 is the number of wires in the wire batch. D_{GMR} takes the place of r' in equation 2.

Geometric Mean Distance (GMD): In situations with more than two wires carrying current (i.e. three phases), for each wire we use D_{GMD} to given an "effective distance" between the wire of interest and a fictitious second wire that behaves equivalently to the batch of other wires. For D_{GMD}, N in equation 3 is the number of wires in the sending conductor batch (could be 1), and M in equation 3 is the number of return conductors. D_{GMD} takes the place of D in equation 2.

Using D_{GMR} and D_{GMD}, the equation for the inductance of a given wire (phase) is:

$$L = \frac{\lambda}{I} = 2 \times 10^{-7} \ln \frac{D_{\text{GMD}}}{D_{\text{GMR}}} \tag{4}$$

Q. *Consider a 3 phase wire configuration in which the three phases are arranged in an equilateral triangle configuration, distance D apart. What is the geometric mean of the distances between a single phase and the other two phases?*

In practice, GMR and GMD values are provided for commonly used bundles.

A Derivation of Series Inductance in Transmission Lines

To calculate inductance for a transmission line we must determine the relationship between the current flowing through the line and the flux linkage over all space. Sounds easy. Fortunately, we can use the geometry of the problem and some assumptions to simplify the calculation. The assumptions we use are:

1. The line is infinitely long, and thus end-affects can be neglected and the problem can be addressed in two dimensions rather than three.

2. The current is distributed evenly in the wire (skin-affects are neglected).

3. The permeabilities of the wire, air, and all other materials present are equal to the permeability of free space: $\mu \approx \mu_0 = 4\pi \times 10^{-7}$ H/m.

The magnetic field strength around a given loop is determined by I_{encl}, the current that is enclosed by the loop (Ampere's law):

$$\oint H_{\tan}\, dl = I_{\text{encl}} \tag{5}$$

The flux density is determined by the product of the magnetic permeability of the air and the magnetic field strength. Flux linkage is a geometric property that describes how much flux is "linked" to 1 A of current flowing through a given wire. For a single wire without any coils, the flux linkage is calculated by taking the integral of the flux density over all space. "All space" can be simplified by assuming that the line is infinitely long and focusing on per-unit-length quantities, which allows us to just integrate over a cross-sectional plane. The cross-sectional plane can be further simplified using the radial symmetry—we can integrate the flux density over just the radial distance from the wire (one dimension). Thus, to calculate inductance for a single wire, we have to integrate the flux linkage between two radial distances, D_1 and D_2:

$$\lambda_{1,2} = \int_{D_1}^{D_2} \frac{\mu I}{2\pi x} dx \tag{6}$$

Because inductance is the relationship between line current and flux linkage over all space, we want $\lambda_{0,\infty}$. Note, as we get close to 0, we move inside the wire, and then the enclosed current I starts to change (I goes to 0 as x goes to zero). Therefore it makes sense to break the integral up into the sum of two integrals, one from 0 to the radius r of the conductor, and one from r to ∞. Because we assumed that current is constant in the wire, we can actually combine the two integrals by adjusting where the outside-the-wire integral starts from. Instead of starting the integral at r, we start the integral at r'. $r' = e^{\frac{-1}{4}} r = 0.7788r$—it's a little less than r, so the integral starting from r' will be a bit bigger than the integral starting at r. The integral is bigger because we are now including the flux density that is *inside* the wire in the outside-the-wire integral.

The outside-the-wire integral is integrated from r' to infinity. And beyond. Fortunately, we can exploit the symmetry *again*. Transmission lines will always have a return path—current flows in loops so the current that goes out must always come back. The magnetic field strength is given by I_{encl}, the current that is enclosed by a given loop in space (this "loop in space" is *different* than the loop that is created by the current-carrying wire). Exploiting symmetry, we choose the loop in space to be a ring of radius ρ around the wire that we are calculating inductance for. As ρ expands, eventually, the ring will *also* enclose the return path wire. Once the ring includes both the transmission wire and its return path, $I_{\text{encl}} = 0$. If D is the distance between the transmission wire and its return path wire, by equation 5, the integral of the magnetic field around a $\rho > D$ ring will be zero. Thus, for $\rho > D$, there will not be any flux linked to the current in the wire.

Recall that the inductance for a transmission line is given by the relationship between the current flowing in the line and the flux linkage over all space. Using symmetry, we simplified this relationship to the integral of flux density from a radial distance of 0 to ∞. Then, in the previous two paragraphs, we further simplified the integral by determining two new bounds for integration. Instead of integrating the flux density from a radial distance of 0 to ∞, we calculate the flux linkage by integrating the flux density from a radial distance of r' to D:

$$\lambda := \lambda_{0,\infty} = \int_{r'}^{D} \frac{\mu_0 I}{2\pi x} dx = \frac{\mu_0 I}{2\pi} \ln \frac{D}{r'} = 2 \times 10^{-7} I \ln \frac{D}{r'} \tag{7}$$

The equation for inductance L is then:

$$L = \frac{\lambda}{I} = 2 \times 10^{-7} \ln \frac{D}{r'} \tag{8}$$

This is the equation that you need to know for this class.