Introduction to Electric Power Systems
Lecture 6A
Transmission Line Modelling

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1 Lumped RLC components of transmission lines

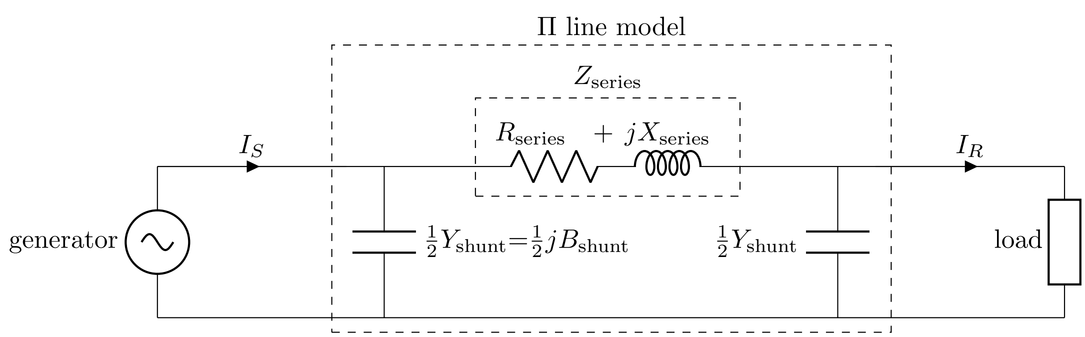

Figure 1: Single load circuit demonstrating lumped series and shunt elements for a Π network model.

Definitions:

- **Series elements**: the circuit components that exist on the current path from the generator to the load and back which include the line, the load, and the return path.
- **Series current**: the current that flows through the series elements.
- **Shunt components**: the circuit components that provide a current path that is parallel to the series element paths. e.g. reactive power compensation shunt capacitor banks are attached to the power lines on one end and the ground on the other end.
- **Shunt current**: current that flows through shunt components.

1.1 Series Resistance

The series resistance describes the resistance of the transmission line. The per-unit-length resistance is a property of the transmission line material and the geometry of the cross-section of a give line, but not on the relationsship between lines.

1.2 Series Inductance

The inductance of a loop of current is determined by the size of the loop. This is because the magnetic flux lines align inside of the loop, and cancel outside of the loop, because of the right-hand-rule. This same size-of-the-loop intuition applies to transmission lines—when the transmission line and its return path are further apart (r is small and D is big), the loop created by the transmission line will be bigger. Note, in power systems, we are interested in inductance per-unit-length of transmission wire. Thus, the *only* dimension that we care about is the cross-sectional distance between the wires, not length of the wires that make the loop.

1.3 Shunt Capacitance

Generally, it is acceptable in power system analysis to take "shunt capacitance" at face value—to accept that there is a parallel current path to ground that can be modeled as a shunt capacitor (recall that with capacitors, charge builds up on the capacitor plate, but no electrons/charge particles flow across). The physical explanation of this parallel path to ground is that when the voltage increases or decreases on a given wire, the charge particles on that wire will be attracted to the side of the wire that is closer to ground, or to another wire with a different voltage. When current is flowing through the wire, this attraction to the sides of the wire creates a charge particle "sticking affect." While this affect is small for a small section of wire, the affect adds up over the full length of the wire, and often needs to be taken into account. In the Π line model, the charge-sticking affect is taken into account by placing shunt capacitors on either side of the series line impedance.

1.4 Shunt Conductance

The electrons/charge particles flowing through the air from one line to another line, or to ground. Air is a very good insulator, and therefore pretty much no current flows through the air separating each line from one another and the ground.

Q. *For each of the properties of power lines, describe the physical phenomena or process that create the properties and number them from most to least significant, so that (1) is most significant and (4) is least.*

a) Series resistance:

b) Series inductance:

c) Shunt capacitance

d) Shunt conductance:

Q. *Why is shunt capacitance is more significant for underground lines?*

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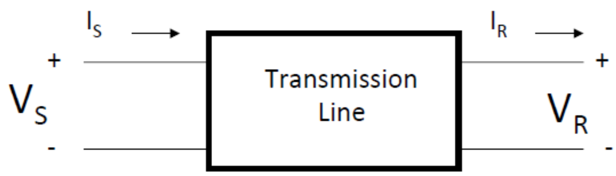
2 Two-port Models

The two port model allows us to represent the Π-modeled transmission line with lumped circuit components in fig. 1, potentially with shunt admittances, using two standard linear equations:



$$\text{With} \quad \begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$

The two-port model equations take into account KCL at both junction nodes and Ohm's law across the series line impedance of the Π-modeled transmission line in fig. 1 to produce the following formulas for the A, B, C, and D matrix entries:

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \left(1 + \frac{YZ}{2}\right) & Z \\ Y\left(1 + \frac{YZ}{4}\right) & \left(1 + \frac{YZ}{2}\right) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Knowing these formulas can make it easier to analyze circuits with shunt admittances.

Q. *Demonstrate that KCL and KVL hold for the equation above by multiplying out the matrix*

Q. *What two equations result when $Y = 0$?*

3 Short, Medium, and Long Line Approximations

So far we have analyzed the **Medium Line Model**, which approximates the traveling wave behavior with lumped components and includes shunt admittances. The **Short Line Model** approximates the Medium Line Model by ignoring the shunt capacitance on the network, and only considering the series impedance.

Alternatively, the **Long Line Model** takes into account the fact that the series inductance, series resistance, and shunt capacitance are "distributed" all the way down the line. Considering this distributed nature allows the Long Line Model to describe the voltage and current at every point along the line, rather than just at the end points. Furthemore, the Long Line Model describes how the voltage and current waveforms "travel" through the network, as described in Appendix A.

Q. *Which of the above properties are captured by each of the circuit models? Sketch the circuit model and label the properties.*

Lossless line:

Short line:

Medium line:

Long line:

Q. *Why is the lossless line approximation more appropriate for transmission than distribution lines?*

Q. *Suppose we drew a circuit model with shunt conductance or capacitance only at the sending or receiving end (not split into both). What important property of a transmission line does this violate? Why does it make sense to require this property physically?*

A *Bonus Material:* Traveling wave interpretation for the Long Line Model

To understand the "traveling wave" interpretation for power systems, you have to remember that the complex numbers used to represent the voltage and current waves are phasor representation, and the phasor representation has discarded the time domain portion of the signal.

The "traveling wave" in the phasor domain is represented by a spatially-varying phase shift. When you step out of the phasor representation back into the time domain, the spatially-varying phase shift will manifest as a wave that "travels" from one direction of the line to another—at one instant the peak of the wave is at one location, and at the next instant the peak has moved to the right or the left. (The traveling direction depends on the sign of the phase shift in the phasor domain.)

The speed of the traveling wave is determined by wave propagation formula:

$$v = f\lambda,$$

where v is the wave speed, f is the frequency, and λ is the wavelength.

The voltage and current waves on power grids "travel" down the lines. At one instant the peak of the voltage and current waves are at one location, and at the next instant each peak has moved down the line. The traveling voltage and current waves are often quite long, though. A standard wavelength for transmission lines, determined by the series inductance and shunt capacitance of the line, is $3000m$. At 60 Hz, the wave speed will be 180 km/s, which is 0.06% percent of the speed of light.

The Long Line Models capture the traveling wave behavior, and the per-unit-length series inductance and shunt capacitance quantities used to determine the Long Line Model are the same per-unit-length series inductance and shunt capacitance quantities that are used to calculate wavelength.

However power systems are rarely analyzed at this level of complexity. In practice, we often use either the Short or Medium Line Models, which approximate the traveling wave behavior with discrete, ideal components. With ideal components, the voltage is constant for each line, and the current is constant for each current path, changing only at junction points.