Introduction to Electric Power Systems
Lecture 5
Harmonics

Keith Moffat

Contents

1 Harmonics **1**
1.1 Linear and Nonlinear Loads . . . 1
1.2 Fourier Series . . . 2
1.3 Total Harmonic Distortion . . . 2
1.4 Harmonics and Real Power . . . 3
1.5 True Power Factor . . . 4

1 Harmonics

For a given frequency f, harmonics are signals at frequencies that are an integer-multiple of f, i.e. a signal at $2f$ or a signal at $3f$. Harmonic currents are created by nonlinear loads. When the current harmonics flow through the network (impedance), the current harmonics create voltage harmonics. As a first order approximation, we ignore voltage harmonics. If the voltage wave does not have harmonics, the current harmonics do not transmit any real power. Thus harmonic distortion is included with reactive power when determining the "true" power factor. This section explains why these things are true.

1.1 Linear and Nonlinear Loads

For our purposes, in AC power systems, a linear element is something that gives a sinusoidal current at the same frequency as the voltage when we apply a sinusoidal voltage to it. By definition, linear elements can't cause harmonics.

Q. *What do we mean by a linear load, or even more generally, by a linear circuit element? What are some examples of linear circuit elements?*

Q. *What are some examples of non-linear elements? What are some loads or power sources that you would find them in?*

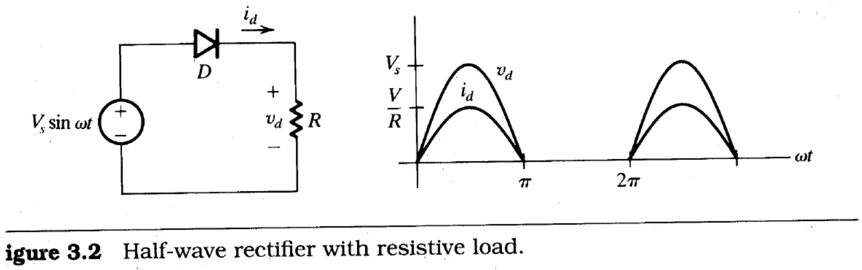

Figure 1: A diode rectifier circuit and corresponding voltage and current waveforms

Fig. 1 gives an example of a rectifier circuit (called a half-wave rectifier) that you would likely study in a power electronics course. Note the non-sinusoidal voltage and current waveforms.

1.2 Fourier Series

The connection between nonlinear loads, nonsinusoidal current waves and harmonics is easiest to understand if we first take some time to understand Fourier Series. The Fourier Series is a mathematical technique that allows you to construct any *periodic* signal with frequency f from a (potentially infinite) sum of harmonics of f.[1]

A nonlinear load will create a nonsinusoidal current wave. The nonsinusoidal current wave will contain harmonics, as described by the Fourier Series decomposition of the nonsinusoidal current wave. That is, whatever load current shape may be created by a nonlinear load, we can describe that load current as the sum of a bunch of harmonics of the fundamental frequency. As a general rule, sharp corners in the load current will result in higher frequency harmonics.

Q. *What harmonics are in the signal* $i(t) = 100\cos(\omega t) + 40\cos(3\omega t) + 30\cos(5\omega t)$*?*

1.3 Total Harmonic Distortion

Total Harmonic Distortion (THD) is the ratio of harmonic current magnitudes to the fundamental current magnitude. Because sinusoids of different frequencies are orthogonal,[2] you add the square of the current magnitudes then take the square root, rather than just summing the current magnitudes. Because of how the currents are added, either RMS or peak amplitude magnitudes can be used, though RMS is default. The equation for Total Harmonic Distortion is:

$$\text{THD} = \frac{\sqrt{\sum_{k=2}^{\infty} I_{\text{k}}^2}}{I_1} \cdot 100\%$$

[1] The Fourier *Transform*, on the other hand, is a mathematical technique that allows you to construct *any* signal from an infinite set of (non-harmonic) sine waves. Because we are dealing with periodic waves, we are interested in the Fourier Series, not the Fourier Transform.

[2] Because the average-over-a-period of the product of sine (cosine) waves of different frequencies is always zero, sine waves of different frequencies are called "orthogonal."

Q. *Calculate the total harmonic distortion for the signal* $i(t) = 100\cos(\omega t) + 40\cos(3\omega t) + 30\cos(5\omega t)$.

1.4 Harmonics and Real Power

The kth harmonic current will only transmit real power if the voltage wave also contains a kth harmonic. In power systems, it is common to assume that voltage wave is a perfect sine wave at the fundamental frequency, and does not contain any harmonics. Thus, harmonic currents will not transmit real power. The instantaneous power wave that arises due to the kth harmonic current will oscillate with an average power of zero.

Q. *Can the instantaneous power that arises due to harmonic currents be thought of as reactive power?*

Real power is the average value of the instantaneous power wave. The instantaneous power wave for the kth harmonic current is the product of the kth harmonic current wave and the fundamental frequency voltage wave. The trig identity used to determine the wave created by the product of two sine waves is:

$$\cos(a)\cos(b) = \frac{1}{2}\Big(\cos(a-b) + \cos(a+b)\Big)$$

For instantaneous power waves, ignoring magnitudes, a and b are functions of time: $a(t) = \omega_a t + \alpha$ and $b(t) = \omega_b t + \beta$:

$$\cos(\omega_a t + \alpha)\cos(\omega_b t + \beta) = \frac{1}{2}\Big(\cos\big((\omega_a t + \alpha) - (\omega_b t + \beta)\big) + \cos\big((\omega_a t + \alpha) + (\omega_b t + \beta)\big)\Big)$$

The product of the two cosines is described by the sum of two cosines.

Q. *Prove harmonic currents do not transmit real power.*

We have demonstrated that real power is only transferred by voltage and current waves that are the same frequency. Thus, if the voltage wave does not have harmonics, all of the power from the current harmonics will be reactive power. That is why THD is included with reactive power when determining power factor. When there are nonlinear loads on a system, the total reactive power can be split up into “phase shift-caused reactive power” and “harmonic-caused reactive power.”

Q. *We have seen that phase shift-caused reactive power can be compensated by injecting conjugate reactive power. Can you cancel reactive power "load" at one frequency by injecting harminc reacive power at another frequency?*

1.5 True Power Factor

The "true" power factor must take into account the reactive power at the fundamental frequency, as well as the reactive power "load" at the harmonic frequencies. The following formula for true power factor does this for you.

$$pf_{true} = \frac{P_1}{V_1 I_1} \cdot \frac{1}{\sqrt{1 + (\mathrm{THD}/100)^2}}$$