Introduction to Electric Power Systems
Lecture 4
Unbalanced 3-phase Circuits and Symmetrical Components

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Contents

1 Unbalanced 3-phase Voltage and Current

Before introducing symmetrical components, we will fist look at unbalanced 3-phase circuits. Up to this point, we have considered 3-phase balanced systems. In practice, 3-phase systems are unbalanced. Often, for transmission networks, this imbalance is small and the balanced circuit approximation is appropriate. However, the balanced circuit approximation is not always appropriate. In particular, distribution networks often have imbalances that are significant enough to warrant full 3-phase analysis.

If a 3-phase set of voltage or current phasors is unbalanced, the phasor magnitudes will not be equal and/or the phasors will not be separated by 120°. The unbalance can arise from an unbalanced generator/voltage source, unbalanced transmission line impedance, or unbalanced loads.

Ohm's law for a three phase circuit is the vector-valued equation

$$\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = Z_{ABC} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix},$$

where Z_{ABC} is a 3x3 matrix.

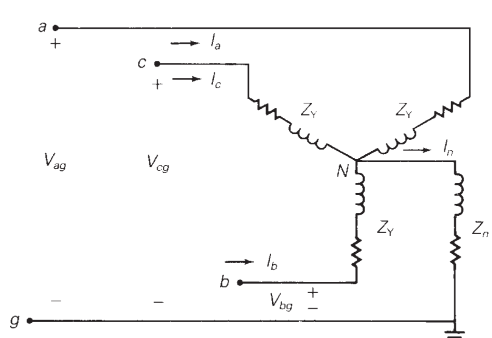


Figure 1: Wye Circuit with Neutral Wire with Impedance

Consider the circuit with the Wye-connected load and neutral line impedance in Fig. 1. Each arm has a constant impedance load Z_Y, and the neutral line impedance is Z_N. While the load impedances are balanced, the circuit will be unbalanced if the voltages applied are unbalanced. Note, *the voltages are phase-to-ground, rather than phase-to-neutral.* The vector-valued Ohm's law can be constructed by applying Ohm's law to each individual phase:

$$\begin{aligned} V_{Ag} &= Z_Y I_A + Z_N I_n \\ &= Z_Y I_A + Z_N(I_A + I_B + I_C) \\ &= (Z_Y + Z_N)I_A + Z_N I_B + Z_N I_C \end{aligned}$$

Combining the Ohm's law equations for each phase gives

$$\begin{bmatrix} V_{Ag} \\ V_{Bg} \\ V_{Cg} \end{bmatrix} = \begin{bmatrix} (Z_Y + Z_N) & Z_N & Z_N \\ Z_N & (Z_Y + Z_N) & Z_N \\ Z_N & Z_N & (Z_Y + Z_N) \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}. \tag{1}$$

The off-diagonal terms of the impedance matrix are referred to as "coupling impedances." Later in the course we will learn that additional inductive and capacitive coupling impedances arise from the physics of the transmission lines.

2 Symmetrical Components

The three phasors of an unbalanced 3-phase signal require six pieces of information, the magnitude and angle of each phase. Symmetrical components is another way of representing the three phasors that constitute a 3-phase voltage or current signal, instead providing the magnitude and angle of the "positive," "negative," and "zero" sequence components. Both the three-phasor representation and the symmetrical components or "sequence" representation of a 3-phase signal have six pieces of information: three magnitudes and three phase-shifts.

Symmetrical components are ubiquitous in power system analysis because symmetrical components make it easier to

1. analyze balanced circuits,
2. analyze unbalanced circuits with balanced transmission lines, and
3. detect unbalanced 3-phase faults.

2.1 Mathematical Derivation/Interpretation

Symmetrical components contain the same information as the canonical 3-phase phasor representation, but in a different coordinate system.[1] The transformation from phase representation to symmetrical components is described by the following matrix multiplication:

$$V_{012} = \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix}, \tag{2}$$
$$\alpha = e^{j\frac{2\pi}{3}} = e^{j120^\circ}.$$

[1] There are other tranforms/coordinate systems used in power engineering which you may encounter when modelling generators or power electronics circuits. The other commonly-used transforms are the direct-quadrature-zero transform (abbreviated *dq0* or *dq*) and the closely-related alpha-beta-zero transform ($\alpha\beta 0$ or $\alpha\beta$), which are applied to time-domain, rather than phasor-domain, signals. Note, the 0 component in *dq0* coordinates is *not* the same thing the 0 sequence for symmetrical components.

The same equation can, of course, be used for current phasors as well:

$$I_{012} = \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}. \tag{3}$$

The inverse transformation is given by the inverse of (2):

$$V_{ABC} = \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} \tag{4}$$

Note, the product of the two transformation matrices gives the identity matrix, as expected.

2.2 Sequence Impedance

Ohm's law for the sequence representation of the phasors is

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = Z_{012} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}.$$

The sequence impedance matrix for the network described by fig. 1 and eqn. (1) is:

$$Z_{012} = \begin{bmatrix} (Z_Y + 3Z_N) & 0 & 0 \\ 0 & Z_Y & 0 \\ 0 & 0 & Z_Y \end{bmatrix}. \tag{5}$$

Q. *What is remarkable about the sequence impedance matrix (5)? What does this tell us about sequence analysis for 3-phase circuits with coupling impedance?*

Q. *Derive the equation for the sequence impedance (5) using (1), (2), and (3).*

Q. *What is the positive sequence component of the following 3-phase impedance matrix?*

$$Z_{ABC} = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$$

2.3 Constituent Phasor Interpretation

Consider the following unbalanced 3-phase signal

$$\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} .8\angle -18^\circ \\ 2.3\angle -162^\circ \\ 1.4\angle 117^\circ \end{bmatrix}$$

and its corresponding phasor diagram:

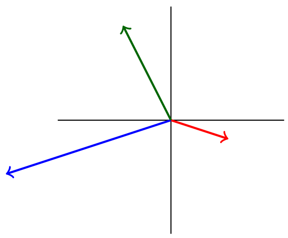
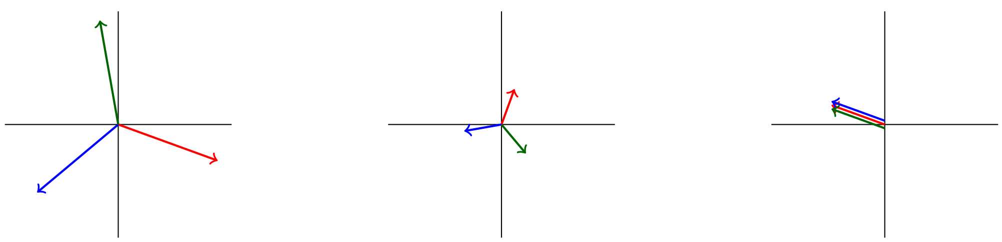


Figure 2: Unabalanced 3-phase signal

Using the transformation in eqn. (2), the three unbalanced phasors described in fig. (2) can be represented as sequence components:

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} .8\angle -18^\circ \\ 2.3\angle -162^\circ \\ 1.4\angle 117^\circ \end{bmatrix} = \begin{bmatrix} .75\angle 160^\circ \\ 1.4\angle -20^\circ \\ 0.5\angle 70^\circ \end{bmatrix}$$

Which correspond to the following sequence components:



(a) Positive Sequence (b) Negative Sequence (c) Zero Sequence

Figure 3: Constituent phasors for the positive, negative and zero symmetrical components

V_0, V_1, and V_2 are the magnitude and phase shift of the A "constituent phasor" for the zero, positive, and negative sequences, respectively. The A constituent phasors for each sequence component are accompanied by B and C constituent phasors in the following manner:

- Positive Sequence: the A, B and C constituent phasors have the same magnitude and are separated by 120° in the order A then B then C.
- Negative Sequence: the A, B and C constituent phasors have the same magnitude and are separated by 120° in the order A then C then B.
- Zero Sequence: the A, B and C constituent phasors have the same magnitude and point in the same direction (are separated by 0°).

Because the magnitudes are equal and the phase shift between the constituent phasors are set for each sequence component (120° or 0°), each sequence component only requires the magnitude and phase shift of the A–phase constituent phasor.

Q. *What are the positive, negative, and zero symmetrical components of a balanced, 3-phase signal?*

Q. *What portion of a set of 3-phase unbalanced current phasors goes through the neutral wire?*

Q. *What portion of an unbalanced three phase voltage or current signal does the positive sequence represent?*

Q. *What portion of an unbalanced three phase voltage or current signal does the negative sequence represent?*

What happens when the negative and positive symmetrical components are equal?

What happens when the negative sequence magnitude is larger than the positive sequence magnitude?

Q. *What portion of an unbalanced three phase voltage or current signal does the zero sequence represent?*

2.4 Symmetrical Component Applications

At the beginning of this section we stated that symmetrical components are ubiquitous in power system analysis because symmetrical components make it easier to

1. analyze balanced circuits,
2. analyze unbalanced circuits with balanced transmission lines, and
3. detect unbalanced 3-phase faults.

Q. *When people refer to the "single phase equivalent" for a balanced, 3-phase circuit, what are they referring to? Why is it easier to analyze a balanced circuit as a single phase equivalent?*

Q. *Why is it generally easier analyze unbalanced circuits with balanced transmission lines in symmetrical components?*

Q. *How can you detect unbalanced faults using symmetrical components?*
