Introduction to Electric Power Systems
Lecture 3
3-phase Power

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Contents

1 Why Balanced 3-phase power?

There are three main reasons. Three is the minimum number of phases that possess all three of these qualities.

Reason 1: Compared with separate single–phase systems, multi–phase systems deliver the same amount of power while requiring half the number of conductors.

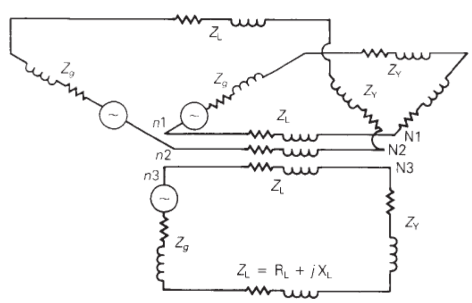

Figure 1: Three Single Phase Power Lines
from p. 69 of GOS

Consider what happens if the voltages are equal and 120° out of phase, the impedances are equal, and if nodes $n1$, $n2$ and $n3$, and nodes $N1$, $N2$ and $N3$ are connected (creating "neutral" points). At each neutral point, the currents from the three phases will cancel. Thus, no current will flow on the wires between the neutral points and those three wires can be eliminated, which saves money. In practice, the neutral points are usually connected with a thin wire that is not rated for full line current.

Q. *How do you produce 3 voltage sources that are 120° out of phase?*

Reason 2: 3-phase power generates a rotating magnetic field on its own.

When a 2–phase voltage wave is applied to a motor at rest, the motor can start turning in either direction. Once it has started, the motor will continue in whichever direction it started in. Thus, a starter winding is used to ensure that the motor starts in the same direction each time. With 3-phase power, starter windings are not necessary because the three phases specify a rotation direction.

Reason 3: 3-phase power delivers constant instantaneous power.

The instantaneous power for phase A connected in Wye will be:

$$\begin{aligned} p_A(t) &= v_{AN}(t) i_A(t) \\ &= \sqrt{2}|V_{AN}| cos(\omega t + \alpha) * \sqrt{2}|I_A| cos(\omega t + \beta) \\ &= |V_{AN}||I_A| \cos(\alpha - \beta) + |V_{AN}||I_A| \cos(2\omega t + \alpha + \beta) \end{aligned}$$

The instantaneous power wave has a constant (average) term $V_{AN} I_A \cos(\alpha - \beta)$ and a time-varying (wave) term $V_{AN} I_A \cos(2\omega t + \alpha + \beta)$. Because the voltage sources for phases B and C are phase-shifted by 120° ($\frac{2\pi}{3}$) radians), for a balanced load, we know that the instantaneous power waves for phases B and C will be:

$$\begin{aligned} p_B(t) &= |V_{BN}||I_B| \cos(\alpha - \beta) + |V_{BN}||I_B| \cos(2\omega t + \alpha + \beta - \frac{4\pi}{3}) \\ p_C(t) &= |V_{CN}||I_C| \cos(\alpha - \beta) + |V_{CN}||I_B| \cos(2\omega t + \alpha + \beta + \frac{4\pi}{3}) \end{aligned}$$

The instantaneous 3-phase power is the sum of the three phases:

$$p_{3\phi}(t) = p_A(t) + p_B(t) + p_C(t)$$

Because the time-varying (wave) terms for the three different phases are equal magnitude and separated by exactly 120°, the sum of the three will equal zero at every instant. We are left with just the constant terms:

$$p_{3\phi}(t) = 3 * |V_{LN}||I_L| \cos(\alpha - \beta) \tag{1}$$

Thus, the instantaneous 3-phase power is constant.[1]

Q. *Why is constant instantaneous power desirable for motors and generators?*

[1] Note, the instantaneous 3-phase power *can't* be determined by adding the complex S vectors for each phase, because unlike V and I, S is not (technically) a phasor (see Discussion 1). Instead, we determined the 3-phase instantaneous power by adding the instantaneous powers of each of the phases.

Q. *Do 3-phase power systems still have reactive power?*

Q. *What power does each 3-phase power line have to be sized for?*

2 Wye and Delta

There are two different ways to connect the phases of a 3-phase system: Wye and Delta. The phases are connected any time power is injected onto/extracted from the grid. Thus, all 3-phase generators, loads and transformers are attached either in Delta or in Wye. Delta and Wye only define how the lines are connected when they come together, not the lines themselves.[2]

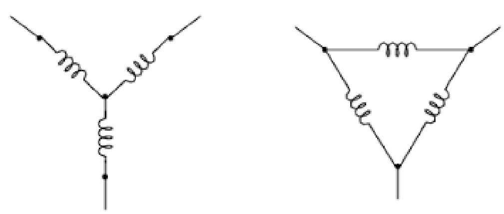

Figure 2: Wye and Delta Connections

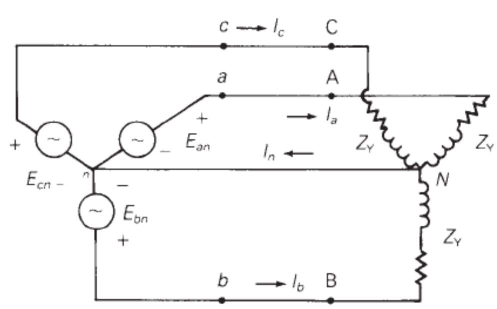

Figure 3: Wye-connected generation and Wye-connected load, with a neutral wire.
from GOS

Transformers can be used to switch from 3-phase wires with a neutral wire to 3-phase wires without a neutral wire:

[2]Often, including a neutral wire is referred to as Wye (because it implies that there are Wye connections on either side), and not including a neutral wire is referred to as Delta. But Wye connections do not necessarily have a neutral line, and therefore it is misleading to refer to lines as "Wye" or "Delta."

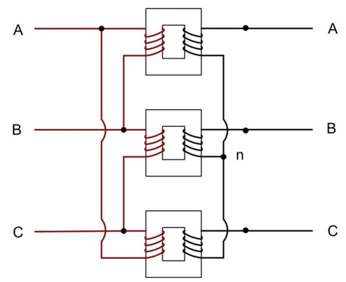

Figure 4: $\Delta - Y$ Transformer
from Wikipedia

Wye connections have:

- Line-to-Line and Line-to-Neutral voltages
- Line currents (but not Line-to-Line currents)

Delta connections have:

- Line-to-Line voltages (but not Line-to-Neutral) voltages
- Line currents and "Line-to-Line" currents

2.1 Line-to-Line Voltages

For a balanced circuit, the Line-to-Neutral voltage waves are shifted by 120° ($\frac{2pi}{3}$). The Line-to-Line voltages are the differences between any two Line-to-Neutral voltages. Thus the Line-to-Line voltages are a sine wave at grid frequency as well, determined by the individual Line-to-Neutral voltages.

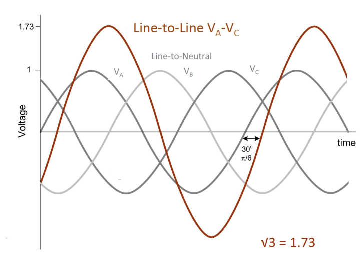

Figure 5: Line-to-Line voltage in time
from the lecture notes

Q. *Using phasors, derive the magnitude and phase shift of the Line-to-Line voltage* V_{AB} *in terms of* $V_{AN} = 1\angle 0$ *and* $V_{BN} = 1\angle -\frac{2pi}{3}$.

Line-to-Neutral and Line-to-Line voltages for a balanced circuit on the same phasor diagram:

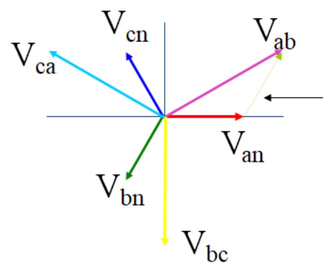

Figure 6: Wye Voltage Phasors
from the lecture notes

Line-to-Line voltages are defined for both Wye connections and Delta connections. The Line-to-Line voltages will always sum to 0. This can be understood using KVL and recognizing that the set of Line-to-Line voltage phasors creates a loop, or with phasors mathematically:

$$V_{AB} + V_{BC} + V_{CA} = V_{AN} - V_{BN} + V_{BN} - V_{CN} + V_{CN} - V_{AN} = 0.$$

2.2 Line-to-Line Currents

The currents on an arm of a Delta connection are Line-to-Line. Defining Line current I_A as positive into node A of the Delta connection, and Line-to-Line current I_{AB} as positive from node A to node B, KCL allows us to write the Line currents in terms of the Line-to-Line currents: $I_A = I_{AB} - I_{CA}$.

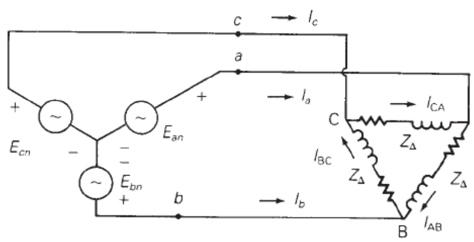

Figure 7: Wye-connected generation and Delta-connected load
from GOS

Thus, to calculate the Line current for a delta load, first you calculate the Line-to-Line currents using the Line-to-Line voltages and Ohms law. Then you calculate the Line currents by adding phasors using KCL, ie $I_A = I_{AB} - I_{CA}$. For a balanced network, the Line currents will be the Line-to-Line currents scaled *up* by $\sqrt{3}$ and shifted *backwards* by 30°.

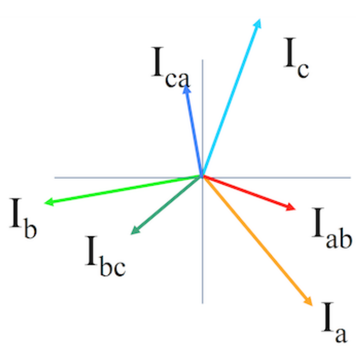

Figure 8: Delta Current Phasors
from the lecture notes

For Delta loads, the *Line* currents must sum to 0, because there is no neutral return. This can be understood by using KCL and treating the entire delta load as a node, or by adding phasors:

$$I_A + I_B + I_C = I_{AB} - I_{CA} + I_{BC} - I_{AB} + I_{CA} - I_{BC} = 0.$$

The sum of *Line* currents also always equals zero for Wye-connected Line currents when the Wye connection does not have a neutral return wire. The sum of Line currents do not always equal zero for Wye-connected Line currents with a neutral return wire. In this case, the sum of the Line currents and the neutral current will equal zero.

3 3-phase Complex Power

Delta and Wye define how the lines are connected when the lines come together. To avoid confusion, we introduce the terminology "arm" to describe the load or transformer windings for either a Wye or Delta connection–both Wye or Delta connections will have three "arms."

- V_{line} refers to a *Line-to-Line* voltage.
- V_{phase} refers to a *Line-to-Neutral* voltage.
- V_{arm} refers to the voltage across a given arm in a Delta or Wye connection. For a Wye connection, it is V_{phase}. For a Delta connection, it is V_{line}.
- I_{line} refers to the current flowing through a given line.
- I_{arm} refers to the current flowing through a given arm in a Delta or Wye connection. For a Wye connection, it is I_{line}. For a Delta connection, it is the linear combination of the line currents on each phase.

For a balanced, three phase Wye connection:

- $I_{\text{line}} = I_{\text{arm}}$
- $V_{\text{line}} = V_{\text{arm}}\sqrt{3}\angle 30°$.

For a balanced, three phase Delta connection:

- $I_{\text{line}} = I_{\text{arm}}\sqrt{3}\angle\text{-}30°$
- $V_{\text{line}} = V_{\text{arm}}$.

The three phase complex power is equal to the sum of the powers in each "phase." For a balanced load, the sum of the powers is equal to three-times the power in one "phase."

Q. *What is the 3-phase complex power in a balanced Wye load, expressed in terms of* V_{LL} *and* I_L*?*

Q. *What is the 3-phase complex power in a balanced Delta load, expressed in terms of* V_{LL} *and* I_L*?*

Thus, for a given voltage and current on a 3-phase power line, the power delivered to a connection is the same, regardless of whether the power is being transmitted to a Delta or Wye connection.

Q. *If you have three resistors, does the amount of power that they dissipate depend on whether you attach them in Delta or Wye? How does the answer to this question correspond to the statement immediately above (that the amount of power is the same for a given voltage and current)?*

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4 Why Wye?

So if you're winding a transformer, when should you use a Wye connection, and when should you use a Delta connection? The primary question isn't really Delta or Wye, but whether or not to include a neutral wire. A Wye connection is necessary to include a neutral wire, but Wye connections do not require a neutral wire.

Q. *Why use a neutral wire?*

Q. *Why not use a neutral wire?*

Q. *If you don't want a neutral wire, should you wind the transformer in Delta or Wye?*

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5 Unbalanced 3-phase Voltage, Current, and Power

Up to this point, we have considered 3-phase balanced systems. In order for a system to be balanced, both the network and the loads must be balanced—the impedances/admittances must be equal for each phase, and the loads must be evenly distributed between each phase. In practice, 3-phase systems (particularly at the distribution level) can and will be unbalanced. In the time domain, unbalanced 3-phase phasors manifest as three waves that have different magnitudes and are not separated by exactly 120°.

Q. *For an unbalanced Wye circuits, will the Line-to-Neutral voltages sum to zero?*

Q. *For an unbalanced Wye circuit, will the Line-to-Line voltages sum to zero?*

Q. *For an unbalanced Delta circuit, will the Line-to-Line voltages sum to zero?*

Q. *For an unbalanced Delta circuit, will the* Line *currents sum to 0?*

Q. *For an unbalanced Delta circuit, will the Line-to-Line currents sum to zero?*

Q. *For unbalanced Wye circuits, will the Line currents sum to zero?*
