Introduction to Electric Power Systems
Lecture 1
Circuit Basics

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Contents

1 Bird's Eye View

Q. *In a sentence or two, what are electric grids?*

Q. *In a sentence or two, why do we have electric grids?*

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2 Energy and Power

Energy: The capacity of something to do work, usually measured in Wh or J ($1J = 1Ws$).

Power: The rate at which energy is transferred from one object to another, usually measured in W. Mathematically, it is the time derivative of energy:

$$P = \frac{dE}{dt}$$

In translational motion, power is measured as force times velocity: Fv.
In rotational motion, power is measured as torque times rotational velocity: $T\omega$.
In electric circuits, power is measured as voltage times current: VI^*.

Mechanical power is converted to electrical power by an Electromotive Force (EMF), usually created by a generator. In the electrical domain, EMF manifests as a voltage.

Q. *What are the two ways that electric power can leave an electric circuit?*

3 Voltage, Current, and Power

Voltage: How much does an electron in one location want to be in another location. Notice that voltage requires *two* locations. Voltage is fundamentally a *difference* or an *across* variable.

Current: The measurement of how many electrons are flowing through an object. Current is a *rate* or a *through* variable.

Power: The product of voltage and current and therefore depends on both how many electrons are flowing and the voltage difference which the electrons are flowing across. Power is *both* an across and a through variable.

Q. *Which current flow represents more power?*

1. 1 electron flowing down a $10V$ differential
2. 10 electrons flowing down a $1V$ differential

The equations that relate power to either voltage or current are nonlinear, whereas the equation that relates voltage to current (Ohm's Law) is linear. Analyzing nonlinear systems of equations is more challenging than analyzing linear systems of equations. Despite this, when analyzing power systems, we usually focus on the nonlinear power equations.

Q. *In the field of power systems, why do we focus on power and not current?*

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4 RMS Voltage and Current

For periodic waveforms, Root-Mean-Squared (RMS) quantities are useful for calculating average power consumption/generation. Thus, in power systems analysis, it is assumed that all V and I amplitude values are RMS quantities:

$$V_{\text{RMS}} = \sqrt{\text{avg}\big(v(t)^2\big)}$$
$$I_{\text{RMS}} = \sqrt{\text{avg}\big(i(t)^2\big)}.$$

Q. *What is V_{RMS} for the sine wave $V\sin(\omega t)$? (Derive the formula even if you know it!)*

Q. *Does $V_{\text{RMS}} = I_{\text{RMS}}Z$ if $v(t) = i(t)Z$? (Ohm's Law)*

Q. *Does the average power $|S_{\text{avg}}| = V_{\text{RMS}}I_{\text{RMS}}$ if $v(t) = i(t)Z$ for some Z?*

Q. *Does $v(t) = i(t)Z$ if $V_{\text{RMS}} = I_{\text{RMS}}Z$?*

Q. *Does $V_{\text{RMS}}I_{\text{RMS}}$ always give $|S_{\text{avg}}|$?*

If $v(t)$ and $i(t)$ are sine waves with the same frequency, then there is always a Z for which $v(t) = i(t)Z$, and we can calculate the average power using $|S_{\text{avg}}| = V_{\text{RMS}}I_{\text{RMS}}$.

Notation: Because RMS quantities are ubiquitous in power system analysis, we will drop the "RMS" subscript from here on, assuming that we are using RMS quantities unless otherwise stated.

5 AC vs. DC power

The electric grid uses AC power, rather than DC power. In AC power, the voltage on the network oscillates in time at the grid frequency (50 or 60Hz). Thus, the current and power on the network will oscillate in time as well. In DC systems the voltage does not oscillate, and thus the current and power do not oscillate. The equation that describes the power that is delivered at a given instant t (instantaneous power) is the same for both AC and DC circuits: $p(t) = v(t)i(t)$.

In general, DC systems are easier to analyze, and have less losses per unit of power transmitted along a transmission line. Yet, our modern power system is AC.

Q. *Why does the electric grid use AC power rather than DC power?*

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6 Phasors

Voltage and current phasors are the complex representation of a snapshot of voltage and current waveforms at a specific frequency f. For power systems, the frequency is almost always the 50 or 60 Hz nominal frequency.

Take the single frequency signal:

$$A\cos(\omega t + \theta)$$

$jA\sin(\omega t + \theta)$ can be added to the signal without changing the real portion of the signal. The original signal can always be retrieved by taking the real portion of the new complex signal:

$$A\cos(\omega t + \theta) = \text{Real}\big(A\cos(\omega t + \theta) + jA\sin(\omega t + \theta)\big)$$

$jA\sin(\omega t + \theta)$ was chosen specifically because it fits into Euler's equation:

$$Ae^{j(\omega t+\theta)} = \big(A\cos(\omega t + \theta) + jA\sin(\omega t + \theta)\big)$$

Thus

$$A\cos(\omega t + \theta) = \text{Real}\big(Ae^{j(\omega t+\theta)}\big)$$

$Ae^{j(\omega t+\theta)}$ can be broken into $Ae^{j\theta}e^{j\omega t}$, and if we remember that we are dealing with AC signals at the frequency ω, we can discard the $e^{j\omega t}$ term. (Formally, this is done with the Fourier transform.) That leaves us with the phasor:

$$Ae^{j\theta}$$

From our knowledge of Euler's formula, we recognize A as the amplitude and θ as the angle of the phasor in the complex plane. The time domain signal can always be retrieved from the phasor by multiplying the phasor by $e^{j\omega t}$ and taking the real portion of the signal.

The two primary ways that we use voltage and current phasors are ohms law and average power calculations.

Ohms law: $V = IZ$

Z is the complex impedance representation of a resistor, inductor or capacitor. Z is not a phasor itself.

Time-average power: $S = VI^*$

S is complex power, and is also not a phasor. This equation is really incredible. We will go into detail on it next week.

Q. *By using complex phasors to represent the AC voltage and current waveforms, what do we not have to solve?*

A *Review Material*: Circuit Components

The grid is a big circuit. The basic circuit components are:

- Voltage/current sources
- Resistors (R)
- Capacitors (C)
- Inductors (L)

Resistors, capacitors, and inductors are "passive" components that do not produce power. At a given instant, a capacitor or inductor might inject power into the circuit, but the capacitor or inductor extracted that power from the circuit at a previous time instant. Voltage/current sources are not passive components, and can inject power into the circuit from an external source.

Generator/Load convention: For generators, the current flows out of the positive voltage. For loads, the current flows into the positive voltage. Note, this is a convention not a fundamental difference, as current can always be negative.

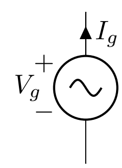


Figure 1: Generator

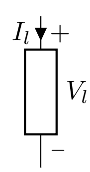
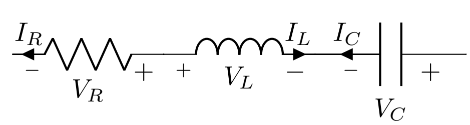
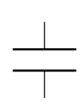


Figure 2: Load

Passive circuit component convention: The current arrow for each component *must* be drawn from the positive terminal to the negative terminal. This rule is required for Ohms law to be consistent with KVL and KCL.



A.1 Capacitors



Capacitors store energy in the form of an electric field between two plates. That electric field is created by a difference in the amount of charge between the two plates. The equation that relates the voltage across the plates to the difference in electrical charge is:

$$q = CV$$

Taking the time derivative of both sides gives the circuit equation for capacitors:

$$i(t) = C\frac{dv(t)}{dt}$$

This is a differential equation. At a specific frequency, the impedance of the capacitor can be represented as a complex number:

$$Z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C}$$

The complex impedance applies to AC voltage and current waveforms represented using phasor notation, which we will introduce next.

Q. *What is the reactance of a 1 mF capacitor at 50 Hz?*

A.2 Inductors

Inductors are the "dual" of capacitors–any law for inductors has an analogous rule for capacitors. Inductors store energy in the form of a magnetic field either in a specifically chosen material, or just the air. The magnetic field (flux) is created by current flowing, according to the right hand rule. This occurs any time electric current flows. When a wire is coiled, the magnetic flux lines align, and the inductance is amplified. This is why coiled wires (usually with a ferromagnetic material in the middle) are called inductors. However a wire does *not* have to be coiled for it to create a magnetic field and have inductance.

The equation that relates the magnetic flux to the electrical current is:

$$\Phi = LI$$

Taking the time derivative of both sides (and using Lenz's Law) gives the circuit equation for inductors:

$$v(t) = L\frac{di(t)}{dt}$$

Once again, this is a differential equation. At a specific frequency, the Impedance of the inductor can be represented as a complex number:

$$Z_L = j\omega L$$

Note, Z_L points in the positive imaginary direction, while Z_C points in the negative imaginary direction. The impedance for a resistor, $Z_R = R$, points in the positive real direction.

B *Review Material*: Complex Arithmetic

Complex numbers are numbers that have both a "real" part and an "imaginary" part, which is multiplied by the imaginary number $j = \sqrt{-1}$.

Q. *What are imaginary numbers?*

Complex numbers can be used to represent signals of a specific frequency in a convenient manner and can be represented in Cartesian coordinates, with a real and imaginary part:

$$Z = R + Xj$$

Complex numbers can also be represented in polar form, using Euler's equation:

$$|Z|e^{j\theta} = |Z|\cos(\theta) + |Z|\sin(\theta)j$$

B.1 Complex Arithmetic exercises

For reference, the following formulas (derived from Euler's equation) convert between polar and Cartesian coordinates.[1]

$$R = |Z|\cos(\theta)$$
$$X = |Z|\sin(\theta)$$
$$|Z| = \sqrt{R^2 + X^2}$$
$$\theta = \tan^{-1}(\frac{X}{R})$$

Q. *Express* $e^{j\frac{7\pi}{6}}$ *in rectangular coordinates. Also write (or draw) its complex conjugate*

Q. *Express* $3 + j6$ *in polar coordinates. Also write (or draw) its complex conjugate*

Q. *Add the quantities from the first two questions above and plot them on the complex plane*

$e^{j\frac{7\pi}{6}} + (3 + j6) =$

[1] if you have a complex denominator, multiply the top and bottom by the complex conjugate to retrieve the desired Cartesian representation of the complex number.

Q. *Multiply the quantities from the first two questions above and plot them on the complex plane*

$e^{j\frac{7\pi}{6}} \times (3 + j6) =$

As you can see, addition is much easier in Cartesian coordinates, while multiplication is easier in Polar coordinates, and more intuitive. Also, complex multiplication performs two transformations, a scaling and a phase shift.

C *Bonus Material*: Phasor notation via the Fourier Transform

In section 6 we derived the phasor representation using our intuition and a few clever tricks, e.g. adding $jA\sin(\omega t + \theta)$ to the real-valued sinusoid to get the complex exponential. The phasor notation can also be derived using more general techniques; specifically by taking the Fourier Transform of the corresponding analytical signal—a common approach in signal processing.

$A\sin(\omega t+\theta)$ is the Hilbert Transform of $A\cos(\omega t+\theta)$. Thus, adding $jA\sin(\omega t+\theta)$ to $A\cos(\omega t+\theta)$ gives an analytical signal with only positive frequency components. (Negative frequency components do not manifest physically for time series waveforms, but must be acknowledged mathematically.) The Fourier transform of the analytical signal $A\cos(\omega t + \theta) + jA\sin(\omega t + \theta) = Ae^{i(\omega t+\theta)}$ is $Ae^{i\theta}$, or the "phasor."
Thus, phasors can be understood through the following steps:

1. Adding $jA\sin(\omega t + \theta)$ (j times the Hilbert Transform of the signal) to make the signal analytic.

2. Taking the Fourier transform.

If the value at a given instant is desired, these steps are conducted in reverse:

1. The inverse Fourier transform reintroduces the time-varying component.

2. Undoing the addition of $jA\sin(\omega t + \theta)$ by taking the real portion of the signal.