

# Introduction to Electric Power Systems

## Lecture 12

# Droop Control

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## 1 Frequency Stability

This week we will talk about frequency stability. Frequency stability is determined by the aggregate power balance (total supply minus total demand) for the network. Thus, frequency stability is a network-wide phenomenon, which is different than voltage or angle stability, which are local phenomena.<sup>1</sup>

Frequency stability is dependent upon operator decisions. It is not safe for machines to stay connected to the grid if the frequency deviates from the nominal frequency too much, due to the mechanical resonant modes of the generator. Thus, generator operators will shut off their generator to avoid damaging their machines. In low frequency scenarios, disconnecting generators exacerbates the power imbalance, resulting in the frequency dropping further. To avoid scenarios in which generator operators detach generators in a cascading manner, grid operators have a handful of tools that help them match supply and demand. The last line of defense for grid operators is “load shedding.” Load shedding is what power system operators call turning off the power by opening the switch that connects a given region of the network (blackouts). To avoid load shedding, it is important to maintain the network frequency close to the nominal frequency.

## 2 Inertia

For an AC grid with rotating machines (generators or motors), grid frequency and the network-wide power balance are inextricably linked by the inertia of the rotating masses—When total supply is greater than total demand, frequency increases, and vice versa, according to the swing equation

$$P_{\text{net}} = \frac{2H}{\omega_s} \ddot{\delta}. \quad (1)$$

The inertia of a rotating machine is physically coupled to the frequency of the grid as the rotor spins in the stator.

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<sup>1</sup>Voltage instability is a local phenomenon related to the maximum power transfer over transmission lines, resulting in the crash in the voltage in a given region of the network. Angle instability is also a local phenomenon related to the maximum power transfer over transmission lines, resulting in the de-synchronization of a given machine—see the lecture on the equal area criterion. Both voltage and angle stability are caused by attaching too much load *in a given region* of the network.

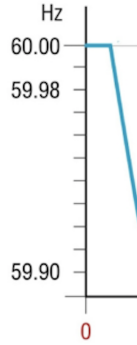


Figure 1: Example of the frequency response on a grid with a power imbalance

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**Q.** Which events could the graph in Fig. 1 be describing?

1. A large increase in load
2. A large decrease in load
3. A large generator tripping offline
4. A large generator coming online

**Q.** What quantity gives you information about the total system inertia?

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### 3 Droop Control

In order for electric power systems to maintain a relatively constant frequency it is necessary that power generation and load are met

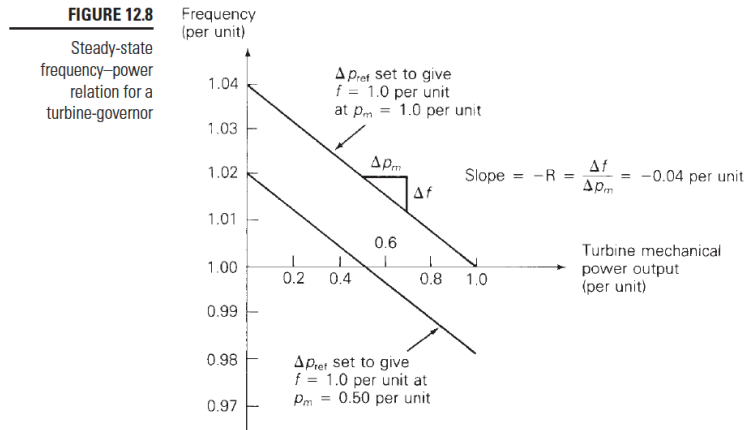
- almost exactly at every moment in time, and
- exactly, averaged over a long time period.

However load on a power system cannot be predicted exactly. So how is it that power systems are able to match a moving load target in real time? Furthermore, the first AC grids were built in the late 1800s, long before computers were invented. The answer is droop control. Before computers, droop control was implemented mechanically, using a generator governor. Today, droop control is still at the heart of stable electric grid operation.

The equation for droop control for a generator  $i$  is given by:

$$\begin{aligned} \Delta f_i &= -R_i \Delta p_{m,i} \\ f - f_{\text{ref},i} &= -R_i (p_{m,i} - p_{\text{ref},i}). \end{aligned} \tag{2}$$

$\Delta f_i$  is the deviation of the frequency  $f$  from generator  $i$ 's reference frequency  $f_{\text{ref},i}$  (usually 50 or 60 Hz),  $R_i$  is generator  $i$ 's droop slope, and  $\Delta p_{m,i}$  is the deviation in generator  $i$ 's mechanical output  $p_{m,i}$  from its reference output  $p_{\text{ref},i}$ .<sup>2</sup>



Droop control is *separate* from the inertia of the system. The inertia of the system, approximated by the swing equation, links the instantaneous power imbalance to the frequency of the generator. Droop control, on the other hand, is an imposed power output rule, not a physical property of the generator. The generator prime mover is modulated so that the generator outputs a certain amount of power for a given network frequency. This relationship is described by Eqn. (2).

**Q.** What level of responsiveness does a steep droop curve (large  $|R|$  indicate)?

**Q.** What level of responsiveness does a flat droop curve (large  $|R|$  indicate)?

**Q.** What is the equivalent slope of a P Q bus that is not running droop?

<sup>2</sup>A quick point on droop convention: Droop curves are typically plotted with frequency on the vertical axis and power on the horizontal axis. In my opinion, it would be more intuitive to put frequency on the horizontal axis and power on the vertical axis. Regardless, we will use the standard convention because you are more likely to see droop described this way.

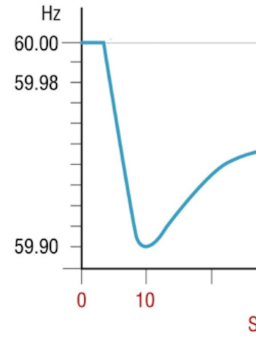


Figure 2: Example of the frequency response on a grid with droop control

**Q.** In Fig. 2, when does the droop control begin to have a significant effect on the frequency?

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**Q.** Suppose I know that this disturbance in Fig. 2 was caused by a 10% increase in load. What must the total system droop coefficient be?

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### 3.1 Droop Steady State Error

One side affect of obeying Eqn. (2) (rather than, for example, an integral control law that returns the frequency to steady state) is that the frequency has steady state frequency error. That is, unless the constants in Eqn. (2) are adjusted, the frequency will not be exactly 50 or 60 Hz. After understanding droop control, we will get into Area Control Error control, which does just that—adjusts the droop constants to eliminate steady state error.

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**Q.** Why is it a good idea for droop control to not include an integrator term?

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### 3.2 Droop Characteristic with Multiple Generators

Often, we are interested in the aggregate droop characteristic for a given area.

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**Q.** *When deriving the aggregate group droop characteristic what term is shared between all of the generators? And what term is added?*

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The aggregate droop characteristic is derived in Ch. 12 of GOS:

$$\begin{aligned}\Delta p_m &= \Delta p_{m1} + \Delta p_{m2} + \Delta p_{m3} + \cdots \\ &= (\Delta p_{\text{ref1}} + \Delta p_{\text{ref2}} + \cdots) - \left( \frac{1}{R_1} + \frac{1}{R_2} + \cdots \right) \Delta f \\ &= \Delta p_{\text{ref}} - \left( \frac{1}{R_1} + \frac{1}{R_2} + \cdots \right) \Delta f\end{aligned}\tag{12.2.2}$$

where  $\Delta p_m$  is the total change in turbine mechanical powers and  $\Delta p_{\text{ref}}$  is the total change in reference power settings within the area. We define the *area frequency response characteristic*  $\beta$  as

$$\beta = \left( \frac{1}{R_1} + \frac{1}{R_2} + \cdots \right)\tag{12.2.3}$$

Using (12.2.3) in (12.2.2),

$$\Delta p_m = \Delta p_{\text{ref}} - \beta \Delta f\tag{12.2.4}$$

Note that, in the convention of the figure above with frequency on the vertical axis, the droop curve gets flatter as more generators running droop are added to the network. This adheres to intuition—as more generators adjust their power output according to the frequency signal, a smaller frequency signal is needed to get the same aggregate power response.